

Exploring personality profiles with matrices

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JOINT MATHEMATICS MEETINGS

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Setting for the project

- **One-semester linear algebra course supporting:**
 - Minor in Mathematics
 - B.S. Game Programming
 - B.S. Acoustics
- **Applications include:**
 - Interpolation
 - Kirchhoff Laws
 - Leslie matrices
 - Stochastic matrices
 - Least squares approximations



Project Goals

- Use technology (not necessarily programming) in an essential way, more than just for row-reduction
- Incorporate the project early in the semester
- Motivate students to anticipate the outcome of the project
- Finish the bulk of the work in a single two-hour class period

Project outline

- Use matrices to model the pairwise compatibility of students in the class, based on their responses to a screening survey.
- Use a test question to measure the success of the compatibility model.
- Solicit student feedback after the project.
- Do the calculations in Microsoft Excel.

Why would I let students use Excel for real math?

- Available in all of our computer labs with no maximum number of simultaneous users
- Free (or at least Google Sheets is)
- Easy to use (students are already familiar with basic use)
- Surprisingly robust, and common in the world of applied mathematics

Preparatory work

- Brief lesson on symmetric matrices (10 minutes)
- Practice matrix operations with an Excel workbook (20 minutes)

Basic compatibility

Example

Consider a matrix representing distances between points

Transpose

Definition

Matrix Products

Matrix products are calculated as a series of dot products.

Multiplying a pair of matrices

Let A be an $m \times n$ matrix, and \vec{a}_i be the i th row of A . Similarly, let B be an $n \times r$ matrix, and \vec{b}_j be the j th column of B . Then their product AB is a matrix C , whose entries are determined as follows (recall that c_{ij} is the entry in the i th row and j th column of the matrix C):

$$c_{ij} = \vec{a}_i \cdot \vec{b}_j$$

Loosely, dot the rows of A with the columns of B .

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Make this page look nice: resize the grids to make them square.														
2															
3	Multiply matrices A and B below; place the result in the box for AB.														
4	Matrix A			Matrix B			Matrix AB								
5	-2	4	3	5	0										
6	0	2	5	-5	4										
7				2	3										
8															
9	Try it again, this time using the names "MatrixA" and "MatrixB."														
10	Matrix A			Matrix B			Matrix AB								
11	-2	4	3	5	0										
12	0	2	5	-5	4										
13				2	3										
14															
15	Take the transpose of Matrix C below, placing it in the boxed-in area.														
16	Matrix C			C Transpose											
17	1	2	3												
18	3	4	5												
19	1	5	-2												
20	0	0	6												
21															

Project structure

- **Using a Moodle-enabled database tool, students answer several Likert-type questions about cultural topics**
 - e.g., “How often do you ride a bike?”, “How closely do you follow sports?”, “How often did you visit art museums in 2015?”
- **Students also answer several open-ended questions**
 - e.g., favorite movies and music
- **Using compatibility matrix techniques, students found their best/worst matches from the Likert questions, and evaluated their matches by comparing their answers to the open-ended questions**
- **Students reported their results and wrote an evaluation of the lesson**

Compatibility matrix

Arrange a list of responses to a yes/no survey in a matrix.

	Q1	Q2	Q3	Q4	Q5	Q6
p_1	N	Y	Y	N	Y	N
p_2	Y	Y	N	N	Y	Y
p_3	N	N	N	Y	Y	N
p_4	Y	Y	N	N	Y	N
p_5	N	Y	Y	N	Y	Y
p_6	Y	Y	N	N	Y	Y
p_7	N	Y	N	Y	N	Y

Compare the respondents to return a symmetric matrix X so that entry X_{ij} assigns a compatibility rating between respondent p_i and respondent p_j .

Method 1: Positive preferences

Assign N a value of 0, and Y a value of 1 to form the “positive-preference matrix” X_P .

	Q1	Q2	Q3	Q4	Q5	Q6
p_1	0	1	1	0	1	0
p_2	1	1	0	0	1	1
p_3	0	0	0	1	1	0
p_4	1	1	0	0	1	0
p_5	0	1	1	0	1	1
p_6	1	1	0	0	1	1
p_7	0	1	0	1	0	1

Method 1: Positive preferences

Multiplying $X_p X_p^T$ yields a symmetric matrix in which each entry x_{ij} represents the number of “yes” values in common between person i and person j .

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_1	3	2	1	2	3	2	1
p_2	2	4	1	3	3	4	2
p_3	1	1	2	1	1	1	1
p_4	2	3	1	3	2	3	1
p_5	3	3	1	2	4	3	2
p_6	2	4	1	3	3	4	2
p_7	1	2	1	1	2	2	3

Method 2: Positive-negative preferences

Assign N a value of -1, and Y a value of 1 to form the “positive-negative preference matrix” X_{PN} .

	Q1	Q2	Q3	Q4	Q5	Q6
p_1	-1	1	1	-1	1	-1
p_2	1	1	-1	-1	1	1
p_3	-1	-1	-1	1	1	-1
p_4	1	1	-1	-1	1	-1
p_5	-1	1	1	-1	1	1
p_6	1	1	-1	-1	1	1
p_7	-1	1	-1	1	-1	1

Method 2: Positive-negative preferences

Again, multiplying $X_{PN}X_{PN}^T$ yields a symmetric matrix; this time, x_{ij} represents, between persons i and j , the number of their agreements less the number of their disagreements.

$$\begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{array} \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 \\ 6 & 0 & 0 & 2 & 4 & 0 & -2 \\ 0 & 6 & -2 & 4 & 2 & 6 & 0 \\ 0 & -2 & 6 & 0 & -2 & -2 & 0 \\ 2 & 4 & 0 & 6 & 0 & 4 & -2 \\ 4 & 2 & -2 & 0 & 6 & 2 & 0 \\ 0 & 6 & -2 & 4 & 2 & 6 & 0 \\ -2 & 0 & 0 & -2 & 0 & 0 & 6 \end{bmatrix}$$

Compatibility through dot product

Positive-preferences



Positive agreement in 3 out of 7 categories: expect some common ground

Positive-negative-preferences



Disagreement in 7 of 7 categories: no common ground at all

Results

	Acosta, Alejandro	Brown, Kendall	Choi, Julian	Garcia, Isabel	Hong, Minho	Howard, Carter	Lerblum, Wesley	Lofgren, Drew	Shaw, Christopher	Smith, David	Viras, Jeremy	Zheng, Sihe
Acosta, Alejandro	5	2	2	1	1	2	4	3	3	5	3	3
Brown, Kendall	2	2	2	1	1	1	2	2	1	2	1	2
Choi, Julian	2	2	2	1	1	1	2	2	1	2	1	2
Garcia, Isabel	1	1	1	2	1	2	1	1	2	2	2	1
Hong, Minho	1	1	1	1	1	1	1	1	1	1	1	1
Howard, Carter	2	1	1	2	1	3	2	1	3	3	3	1
Lerblum, Wesley	4	2	2	1	1	2	4	3	2	4	3	2
Lofgren, Drew	3	2	2	1	1	1	3	3	1	3	2	2
Shaw, Christopher	3	1	1	2	1	3	2	1	4	4	3	2
Smith, David	5	2	2	2	1	3	4	3	4	6	4	3
Viras, Jeremy	3	1	1	2	1	3	3	2	3	4	4	1
Zheng, Sihe	3	2	2	1	1	1	2	2	2	3	1	3

Positive preference matrix

	Acosta, Alejandro	Brown, Kendall	Choi, Julian	Garcia, Isabel	Hong, Minho	Howard, Carter	Lerblum, Wesley	Lofgren, Drew	Shaw, Christopher	Smith, David	Viras, Jeremy	Zheng, Sihe
Acosta, Alejandro	6	0	0	-4	-2	-2	4	2	0	4	0	2
Brown, Kendall	0	6	6	2	4	0	2	4	-2	-2	-2	4
Choi, Julian	0	6	6	2	4	0	2	4	-2	-2	-2	4
Garcia, Isabel	-4	2	2	6	4	4	-2	0	2	-2	2	0
Hong, Minho	-2	4	4	4	6	2	0	2	0	-4	0	2
Howard, Carter	-2	0	0	4	2	6	0	-2	4	0	4	-2
Lerblum, Wesley	4	2	2	-2	0	0	6	4	-2	2	2	0
Lofgren, Drew	2	4	4	0	2	-2	4	6	-4	0	0	2
Shaw, Christopher	0	-2	-2	2	0	4	-2	-4	6	2	2	0
Smith, David	4	-2	-2	-2	-4	0	2	0	2	6	2	0
Viras, Jeremy	0	-2	-2	2	0	4	2	0	2	2	6	-4
Zheng, Sihe	2	4	4	0	2	-2	0	2	0	0	-4	6

Positive-negative preference matrix

Student feedback

Prompt: "Share your thoughts about how this project went today."

- *I liked it. Learning the power of excel is very useful I think, even if we only use it this once in this class.*
- *It was very interesting and I learned a bit more about my class.*
- *Very cool but I think a different set of questions regarding musical tastes and film tastes would yield more relevant correlations*
- *Interesting way to work with matrices*
- *I like this project actually, really helpful to learn about how to use the excel to build the matrix. Somehow I feel the last two question on the survey is not important for this project.*
- *It was interesting to see matrix operations having "real life" uses.*

Thank you.

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