# Exploring personality profiles with matrices

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JOINT MATHEMATICS MEETINGS

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# Setting for the project

- One-semester linear algebra course supporting:
  - Minor in Mathematics
  - B.S. Game Programming
  - B.S. Acoustics

- Applications include:
  - Interpolation
  - Kirchoff Laws
  - Leslie matrices
  - Stochastic matrices
  - Least squares approximations

# Project Goals

- Use technology (not necessarily programming) in an essential way, more than just for row-reduction
- Incorporate the project early in the semester
- Motivate students to anticipate the outcome of the project
- Finish the bulk of the work in a single two-hour class period

# Project outline

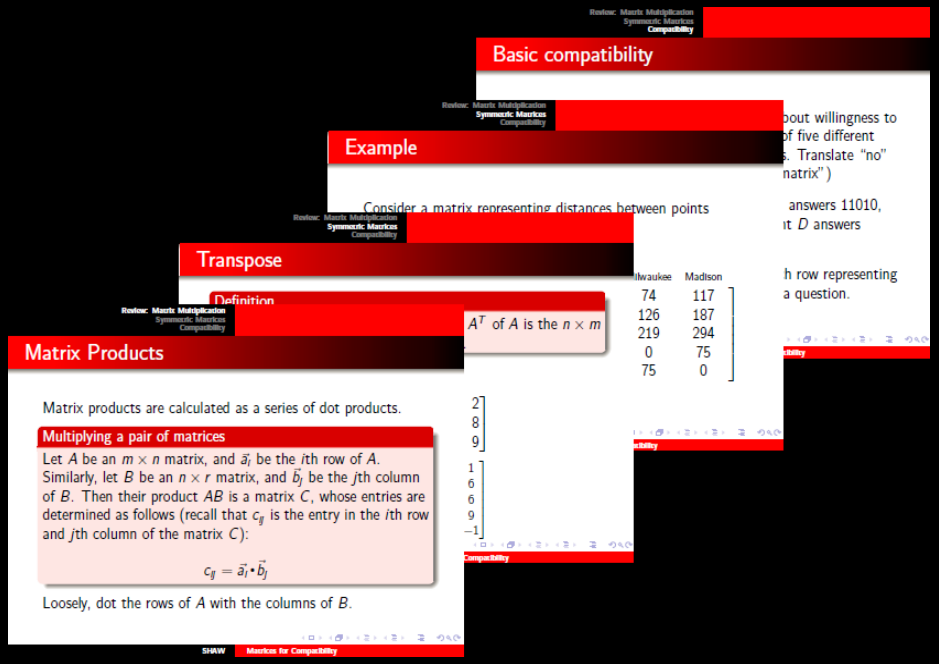
- Use matrices to model the pairwise compatibility of students in the class, based on their responses to a screening survey.
- Use a test question to measure the success of the compatibility model.
- Solicit student feedback after the project.
- Do the calculations in Microsoft Excel.

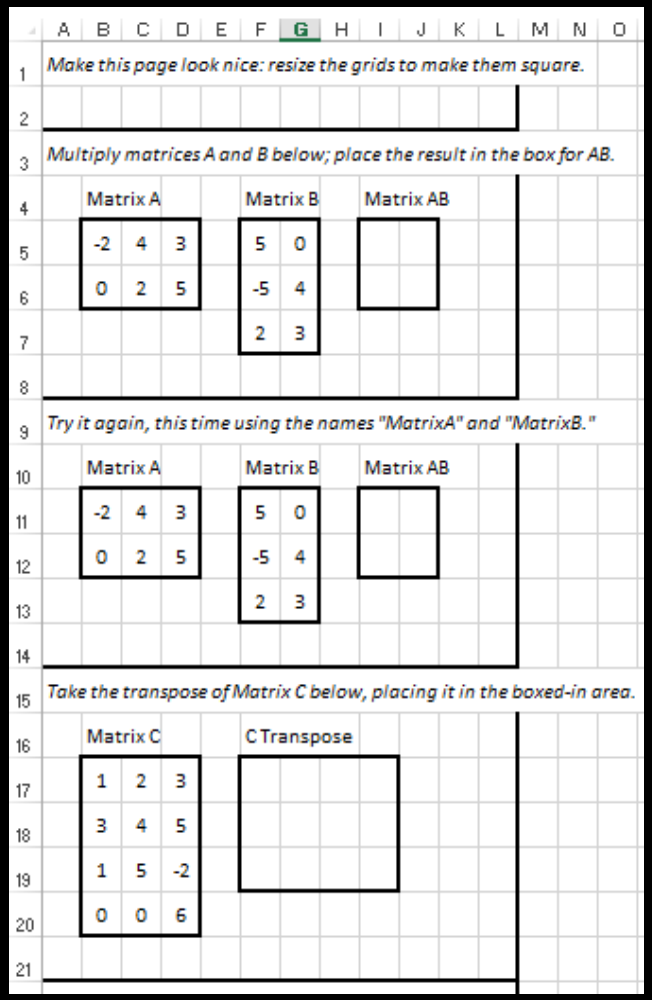
# Why would I let students use Excel for real math?

- Available in all of our computer labs with no maximum number of simultaneous users
- Free (or at least Google Sheets is)
- Easy to use (students are already familiar with basic use)
- Surprisingly robust, and common in the world of applied mathematics

# Preparatory work

- Brief lesson on symmetric matrices (10 minutes)
- Practice matrix operations with an Excel workbook (20 minutes)



# Project structure

- Using a Moodle-enabled database tool, students answer several Likert-type questions about cultural topics
  - e.g., “How often do you ride a bike?”, “How closely do you follow sports?”, “How often did you visit art museums in 2015?”
- Students also answer several open-ended questions
  - e.g., favorite movies and music
- Using compatibility matrix techniques, students found their best/worst matches from the Likert questions, and evaluated their matches by comparing their answers to the open-ended questions
- Students reported their results and wrote an evaluation of the lesson

# Compatibility matrix

Arrange a list of responses to a yes/no survey in a matrix.

| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 |
|---|---|---|---|---|---|---|
| $p_1$ | N | Y | Y | N | Y | N |
| $p_2$ | Y | Y | N | N | Y | Y |
| $p_3$ | N | N | N | Y | Y | N |
| $p_4$ | Y | Y | N | N | Y | N |
| $p_5$ | N | Y | Y | N | Y | Y |
| $p_6$ | Y | Y | N | N | Y | Y |
| $p_7$ | N | Y | N | Y | N | Y |

Compare the respondents to return a symmetric matrix $X$ so that entry $X_{ij}$ assigns a compatibility rating between respondent $p_i$ and respondent $p_j$.

# Method 1: Positive preferences

Assign N a value of 0, and Y a value of 1 to form the "positive-preference matrix" $X_P$.

| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 |
|---|---|---|---|---|---|---|
| $p_1$ | 0 | 1 | 1 | 0 | 1 | 0 |
| $p_2$ | 1 | 1 | 0 | 0 | 1 | 1 |
| $p_3$ | 0 | 0 | 0 | 1 | 1 | 0 |
| $p_4$ | 1 | 1 | 0 | 0 | 1 | 0 |
| $p_5$ | 0 | 1 | 1 | 0 | 1 | 1 |
| $p_6$ | 1 | 1 | 0 | 0 | 1 | 1 |
| $p_7$ | 0 | 1 | 0 | 1 | 0 | 1 |

# Method 1: Positive preferences

Multiplying $X_P X_P^T$ yields a symmetric matrix in which each entry $x_{ij}$ represents the number of "yes" values in common between person *i* and person *j*.

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ |
|---|---|---|---|---|---|---|---|
| $p_1$ | 3 | 2 | 1 | 2 | 3 | 2 | 1 |
| $p_2$ | 2 | 4 | 1 | 3 | 3 | 4 | 2 |
| $p_3$ | 1 | 1 | 2 | 1 | 1 | 1 | 1 |
| $p_4$ | 2 | 3 | 1 | 3 | 2 | 3 | 1 |
| $p_5$ | 3 | 3 | 1 | 2 | 4 | 3 | 2 |
| $p_6$ | 2 | 4 | 1 | 3 | 3 | 4 | 2 |
| $p_7$ | 1 | 2 | 1 | 1 | 2 | 2 | 3 |

# Method 2: Positive-negative preferences

Assign N a value of -1, and Y a value of 1 to form the "positive-negative preference matrix" $X_{PN}$.

| | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 |
|---|---|---|---|---|---|---|
| $p_1$ | −1 | 1 | 1 | −1 | 1 | −1 |
| $p_2$ | 1 | 1 | −1 | −1 | 1 | 1 |
| $p_3$ | −1 | −1 | −1 | 1 | 1 | −1 |
| $p_4$ | 1 | 1 | −1 | −1 | 1 | −1 |
| $p_5$ | −1 | 1 | 1 | −1 | 1 | 1 |
| $p_6$ | 1 | 1 | −1 | −1 | 1 | 1 |
| $p_7$ | −1 | 1 | −1 | 1 | −1 | 1 |

# Method 2: Positive-negative preferences

Again, multiplying $X_{PN}X_{PN}^T$ yields a symmetric matrix; this time, $x_{ij}$ represents, between persons *i* and *j*, the number of their agreements less the number of their disagreements.

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ |
|---|---|---|---|---|---|---|---|
| $p_1$ | 6 | 0 | 0 | 2 | 4 | 0 | −2 |
| $p_2$ | 0 | 6 | −2 | 4 | 2 | 6 | 0 |
| $p_3$ | 0 | −2 | 6 | 0 | −2 | −2 | 0 |
| $p_4$ | 2 | 4 | 0 | 6 | 0 | 4 | −2 |
| $p_5$ | 4 | 2 | −2 | 0 | 6 | 2 | 0 |
| $p_6$ | 0 | 6 | −2 | 4 | 2 | 6 | 0 |
| $p_7$ | −2 | 0 | 0 | −2 | 0 | 0 | 6 |

# Compatibility through dot product

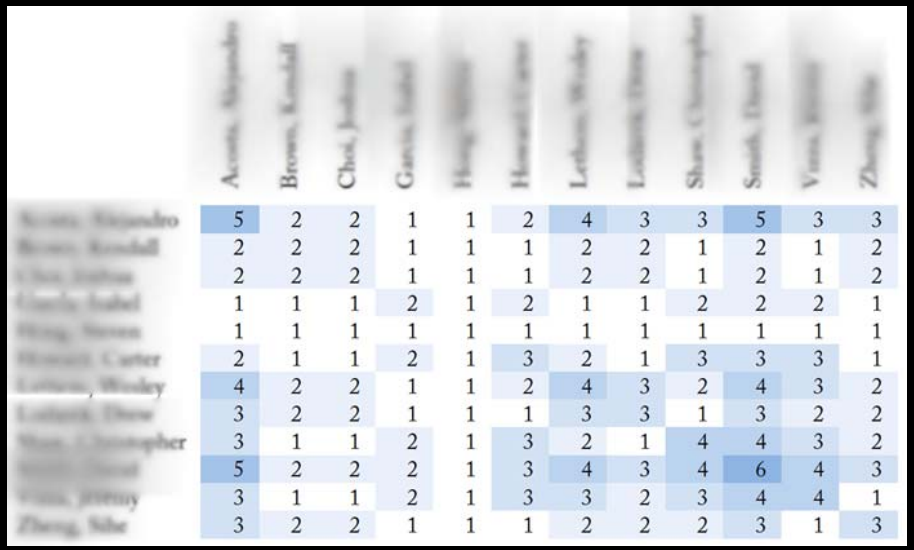
## Positive-preferences


Positive agreement in 3 out of 7 categories: expect some common ground

## Positive-negative-preferences

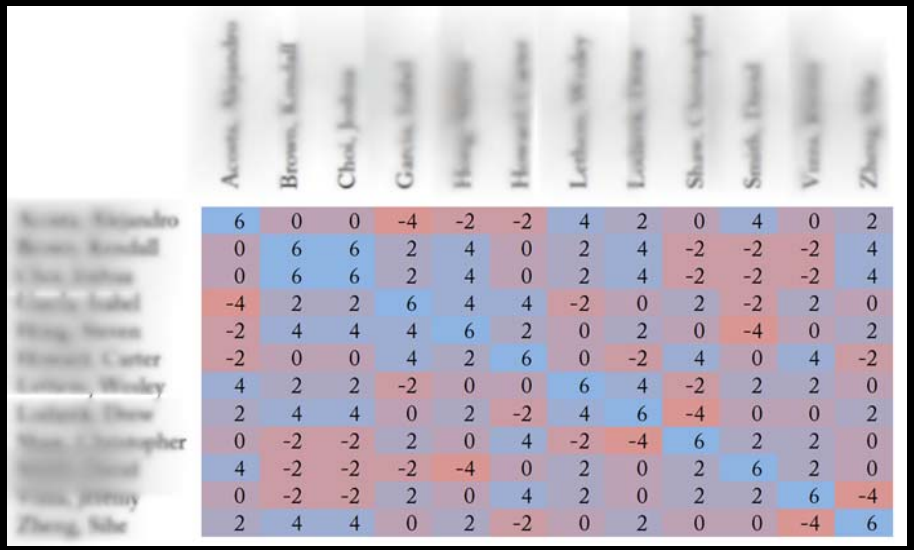

Disagreement in 7 of 7 categories: no common ground at all

# Results



| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [redacted] | 5 | 2 | 2 | 1 | 1 | 2 | 4 | 3 | 3 | 5 | 3 | 3 |
| [redacted] | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 |
| [redacted] | 2 | 2 | 2 | 1 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 |
| [redacted] | 1 | 1 | 1 | 2 | 1 | 2 | 1 | 1 | 2 | 2 | 2 | 1 |
| [redacted] | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| [redacted] | 2 | 1 | 1 | 2 | 1 | 3 | 2 | 1 | 3 | 3 | 3 | 1 |
| [redacted] | 4 | 2 | 2 | 1 | 1 | 2 | 4 | 3 | 2 | 4 | 3 | 2 |
| [redacted] | 3 | 2 | 2 | 1 | 1 | 1 | 3 | 3 | 1 | 3 | 2 | 2 |
| [redacted] | 3 | 1 | 1 | 2 | 1 | 3 | 2 | 1 | 4 | 4 | 3 | 2 |
| [redacted] | 5 | 2 | 2 | 2 | 1 | 3 | 4 | 3 | 4 | 6 | 4 | 3 |
| [redacted] | 3 | 1 | 1 | 2 | 1 | 3 | 3 | 2 | 3 | 4 | 4 | 1 |
| [redacted] | 3 | 2 | 2 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 1 | 3 |

*Positive preference matrix*



| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [redacted] | 6 | 0 | 0 | -4 | -2 | -2 | 4 | 2 | 0 | 4 | 0 | 2 |
| [redacted] | 0 | 6 | 6 | 2 | 4 | 0 | 2 | 4 | -2 | -2 | -2 | 4 |
| [redacted] | 0 | 6 | 6 | 2 | 4 | 0 | 2 | 4 | -2 | -2 | -2 | 4 |
| [redacted] | -4 | 2 | 2 | 6 | 4 | 4 | -2 | 0 | 2 | -2 | 2 | 0 |
| [redacted] | -2 | 4 | 4 | 4 | 6 | 2 | 0 | 2 | 0 | -4 | 0 | 2 |
| [redacted] | -2 | 0 | 0 | 4 | 2 | 6 | 0 | -2 | 4 | 0 | 4 | -2 |
| [redacted] | 4 | 2 | 2 | -2 | 0 | 0 | 6 | 4 | -2 | 2 | 2 | 0 |
| [redacted] | 2 | 4 | 4 | 0 | 2 | -2 | 4 | 6 | -4 | 0 | 0 | 2 |
| [redacted] | 0 | -2 | -2 | 2 | 0 | 4 | -2 | -4 | 6 | 2 | 2 | 0 |
| [redacted] | 4 | -2 | -2 | -2 | -4 | 0 | 2 | 0 | 2 | 6 | 2 | 0 |
| [redacted] | 0 | -2 | -2 | 2 | 0 | 4 | 2 | 0 | 2 | 2 | 6 | -4 |
| [redacted] | 2 | 4 | 4 | 0 | 2 | -2 | 0 | 2 | 0 | 0 | -4 | 6 |

*Positive-negative preference matrix*

# Student feedback

Prompt: “Share your thoughts about how this project went today.”

- *I liked it.  Learning the power of excel is very useful I think, even if we only use it this once in this class.*
- *It was very interesting and I learned a bit more about my class.*
- *Very cool but I think a different set of questions regarding musical tastes and film tastes would yield more relevant correlations*
- *Interesting way to work with matrices*
- *I like this project actually, really helpful to learn about how to use the excel  to build the matrix. Somehow I feel the last two question on the survey is not important for this project.*
- *It was interesting to see matrix operations having "real life" uses.*

# Thank you.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