1. Use logarithmic differentiation to find the derivative:

\[ f(x) = \frac{e^{3x}(x+1)^6(x^3-2)^4}{\sqrt{2x-7}} \]

Recall that logarithmic differentiation relies on the following:

\[ \frac{d}{dx} \left[ \ln(f(x)) \right] = \frac{f'(x)}{f(x)} \]

so

\[ f'(x) = f(x) \cdot \frac{d}{dx} \left[ \ln(f(x)) \right] \]

which is easier to find.

So first we find \( \frac{d}{dx} \left[ \ln(f(x)) \right] \):

\[ \ln \left( \frac{e^{3x}(x+1)^6(x^3-2)^4}{\sqrt{2x-7}} \right) = \ln(e^{3x}) + 6 \ln(x+1) + 4 \ln(x^3-2) - \frac{1}{2} \ln(2x-7) \].

Thus:

\[ f'(x) = f(x) \cdot \frac{d}{dx} \left[ \ln(f(x)) \right] = \left( \frac{e^{3x}(x+1)^6(x^3-2)^4}{\sqrt{2x-7}} \right) \left( 3 + \frac{6}{x+1} + \frac{4 \cdot 3x^2}{x^3-2} - \frac{1}{2} \cdot \frac{2}{2x-7} \right) \]

2. (a) Find all values for \( x \) which make the equation true: \( x \ln(x+1) = 2x \)

To solve, we need to set the summands equal to zero and factor:

\[ x \ln(x+1) - 2x = 0 \]

\[ x(\ln(x+1) - 2) = 0 \]

So, the solutions are \( x = 0 \) and \( \ln(x+1) - 2 = 0 \), which we solve as follows:

\[ \ln(x+1) = 2 \]

\[ e^{\ln(x+1)} = e^2 \]

\[ x + 1 = e^2 \], so:

\[ x = e^2 - 1 \]

(b) Write the following expression in the form \( 2^{ax+b} \) for some \( a \) and \( b \):

\[ \frac{16^x}{2 \cdot 4^{2x}} = \frac{(2^4)^x}{2 \cdot (2^2)^{2x}} = \frac{2^{4x}}{2 \cdot 2^{4x}} = \frac{1}{2} = 2^{-1} \]

3. Find \( g'(1) \), where \( g(x) = e^{3x^2-2x+1} \)

\[ g'(x) = (6x-2) \cdot e^{3x^2-2x+1} \], so \( g'(1) = (6 - 2) \cdot e^{3-2+1} = 4e^2 \]
4. For the following function, determine the **x**- and **y**-value of each critical point, and then use the first or second derivative test to determine whether each point is a maximum, minimum, or neither:

\[ f(x) = \frac{x}{\ln(x)} \]

\[ f'(x) = \frac{1 \cdot \ln(x) - x \cdot \frac{1}{x}}{(\ln(x))^2} = \frac{\ln(x) - 1}{(\ln(x))^2} \]

Solving for zero, we get \( \ln(x) - 1 = 0 \)

\( \ln(x) = 1 \)
\( x = e \)

The first derivative test will have the interval on the left \( x < e \) and the interval on the right \( x > e \). Since the function is not defined for zero or for 1, we will check the value of the derivative for \( \sqrt{e} = e^{\frac{1}{2}} \) and \( e^2 \):

\[ f'(e^{\frac{1}{2}}) = \frac{\frac{1}{2} - 1}{(\frac{1}{2})^2} < 0 \]
\[ f'(e^2) = \frac{2 - 1}{(2)^2} > 0 \]

So by the first derivative test, there is a maximum at \( x = e \), and the **y**-value at this point is \( f(e) = \frac{e}{1} = e \). So the maximum occurs at the point \((e, e)\).

5. To make killer robots, one uses the radioactive isotope **Halloweenium-X**, which has a decay constant of \( \lambda = .002 \).

(a) What is the half-life of **Halloweenium-X**?

Recall that \( L = \frac{\ln(2)}{\lambda} \), so in this case the half-life is \( L = \frac{\ln(2)}{.002} \).

(b) What differential equation is satisfied by the decay of **Halloweenium-X**?

The differential equation for exponential decay is \( P'(t) = -\lambda P(t) \), so \( P'(t) = -.002P(t) \).

(c) Use the differential equation to answer the question: how much **Halloweenium-X** is in a sample which is decaying at the rate of 4 grams per year?

This problem is asking the question: what is \( P(t) \) when \( P'(t) = -4 \)? We answer by plugging the quantity into the equation from part b: \(-4 = - .002 \cdot P(t) \), so \( P(t) = \frac{-4}{-.002} = 2000 \) grams.

6. A colony of flesh-eating zombies feeds on the living population of Earth and grows at a rate proportional to its size. (As with all zombie colonies) it starts with just one person, and after one year there are 2,500 zombies.

(a) Find the growth constant \( k \) of the zombie colony.

The equation is \( P(t) = P_0e^{kt} \), and the problem tells us \( P_0 = 1 \) and \( P(1) = 2500 \). So we solve for \( k \) as follows:

\[ 2500 = 1 \cdot e^{k \cdot 1} \]
\[ \ln(2500) = k \]

(b) Find an equation \( P(t) \) which describes the number of zombies in the colony at time \( t \).

This is straightforward enough. \( P(t) = e^{(\ln(2500)) \cdot t} \)
(c) Using this equation, how many years until the colony reaches 10,000 zombies?
Solve the equation for \( t \):
\[
P(t) = 10000 = 1 \cdot e^{(\ln(2500)) \cdot t}
\]
\[
\ln(10000) = (\ln(2500)) \cdot t
\]
\[
t = \frac{\ln(10000)}{\ln(2500)}
\]

7. An evil collector sells his collection of cursed talismans for \$4,000, deposits the earnings into his bank account, then abruptly disappears and is never heard from again. The account accrues interest compounded continuously for 100 years before someone discovers it, with a balance of \$120,000. What was the annual interest rate for this bank account?
We know that the principal \( P \) is 4000, and we’re given that \( A(100) = 120000 \). So, we solve for \( r \):
\[
A(t) = Pe^{rt}
\]
\[
A(100) = 120000 = 4000e^{r \cdot 100}
\]
\[
\frac{120000}{4000} = 30 = e^{r \cdot 100}
\]
\[
\ln(30) = 100r
\]
\[
r = \frac{\ln(30)}{100}
\]

8. The concentration of a drug in the bloodstream of a patient, \( t \) hours after injection, is given by:
\[
f(t) = 5(e^{-2t} - e^{-2t}) \text{ units}
\]
At what rate is the drug concentration changing after 4 hours?
We just need to find \( f'(4) \), and this is a derivative we should be able to manage.
\[
f'(t) = 5 \cdot (-2e^{-2t} + 2e^{-2t})
\]
\[
f'(4) = 5 \cdot (-2e^{-8} + 2e^{-8})
\]