Differential Equations - Spring 2012 - Midterm Review

Instructions: Your midterm exam will consist of two parts, an in-class portion (Part A), worth 65 points, and a take-home portion (Part B), worth 35 points. Part A will contain seven problems. You must choose six of these to complete in-class, and the remaining problem will be added to Part B, which will be due without exception or excuse at the beginning of class on Tuesday, March 13.

1. Verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equation; then use variation of parameters to find a particular solution of the given nonhomogeneous equation.

$$t(1-t)y'' + t^2y' - ty = 5; \quad y_1 = t; \quad y_2 = e^t$$

2. Solve the given first-order linear IVP.

$$y' - 4y = -e^{-6t}; \quad y(0) = -\frac{1}{2}$$

3. Find the general solution to the given second-order linear homogeneous equations.
   (a) $y'' + 5y' - y = 0$
   (b) $\frac{1}{25}y'' + y = 0$

4. For the given differential equation, (a) find the general solution in implicit form, and (b) find the function $f(t)$ which is a solution to the equation subject to the initial condition $f(0) = -1$. (Be sure to choose the correct branch of the solution.)

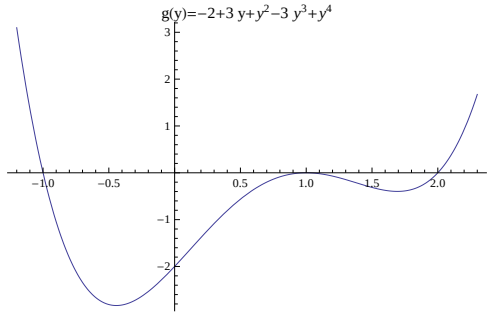
$$4y^3\frac{dy}{dt} = \cos(3t)$$

5. For each of the below equations, (1) determine whether the equation is exact; (2) if it is exact, find the general solution in implicit form.
   (a) $\left(x^2 - 3xy\right)dy + \left(2xy - \frac{3}{2}y^2 + \cos(x)\right)dx = 0$
   (b) $(x^2 + y^3)\frac{dy}{dx} = -(x^3 + y^2)$

6. Use the method of undetermined coefficients to find the particular solution to the differential equation:

$$y'' - 2y' + y = t\cos(t)$$

7. Consider the differential equation $y' = -2 + 3y + y^2 - 3y^3 + y^4$, whose phase curve (the graph of y versus $g(y)$) appears below. Use the phase curve depicted to sketch a graph of several representative solutions $f(t)$ to the differential equation, including each of the equilibrium solutions, as well as solutions corresponding the initial conditions (a) $f(1) = 3$, (b) $f(1) = 1.5$, (c) $f(1) = -.5$, and (d) $f(1) = -2$.



8. A family makes regular deposits into a new account, earning 2% interest, with a starting balance of zero. In the first year, their deposits total \$15,000, and they continue to increase the deposit amount commensurate with inflation. Assuming the deposits occur continuously, and that the annual inflation rate amounts to roughly 3%, write down (but do not solve) an initial value problem that describes the balance in the account t years after opening it.

9. For each matrix below, determine whether it is invertible. If not, explain why. If so, find the inverse.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -6 & -8 \\ 1 & 3 & 6 \end{bmatrix}$$

10. Given $\mathbf{M} = \begin{bmatrix} 1 & i \\ -i & 0 \end{bmatrix}$; $\mathbf{N} = \begin{bmatrix} -2 & 0 \\ 0 & 1+i \end{bmatrix}$, calculate the following:
    (a) $\mathbf{M} + i\mathbf{N}$
    (b) $\mathbf{MN}$

11. Find all eigenvalues for the given matrix, and for each eigenvalue, give the corresponding eigenvectors.

$$\begin{bmatrix} 0 & -2 \\ -1 & -1 \end{bmatrix}$$

12. Consider a spring which, when a 3 lb weight is attached to it, is displaced by one inch. The spring system begins with the weight being pulled down by six inches and released.
    (a) Assuming the spring system to be undamped, find an equation which represents the elongation of the spring from equilibrium, t seconds after the initial release.
    (b) What value of γ, the damping force, would cause this system to exhibit *critical damping*?