Linear v. Nonlinear Differential Equations

The definition of a linear differential equation given in the text on p. 21 is a differential equation of the form

$$F\left(t, y, y', \ldots, y^{(n)}\right) = 0$$

where F is a *linear* function of the variables $y^{(i)}$ with any function of just t allowed to be a coefficient. A way of thinking of this is that the equation can be written as a linear combination of the form

$$\sum_{i=1}^{n} a_i(t) y^{(i)} = g(t)$$

meaning, for instance, that none of the nth derivatives of y may be multiplied by each other or raised to a power - only linear operations of these are allowable.

Examples (linear):

1. $4t^2e^t + 3ty - y'' = 0$
2. $y'''e^t - y = y' + 1$
3. $16y + e^t y'' - \sin(t)y'' + y'' = 10$, because it's equivalent to $16y + \left(e^t - \sin(t) + 1\right) y'' - 10 = 0$
4. $\frac{1}{y'} = 6t^3 - 5$, equivalent to $1 = 6t^3y' - 5y'$, or $6t^3y' - 5y' - 1 = 0$
5. $e^{y'} - 4t = 0$, equivalent to $y' = \ln(4t)$.

Examples (non-linear):

1. $y' = y^2$, because y is being squared, which is not a linear operation
2. $y' \cdot y'' = 4t$, because two derivatives are being multiplied together
3. $\frac{1}{y} - \frac{1}{y'} = 1$, because there is no way to get the y-expressions out of the denominator without ending up with two of them multiplied together.
4. $y' = e^y$, because to release y from the exponent, you would have to take the log of both sides.