Hint for Chapter 2.3, question 1

Despite protestations to the contrary, this question is indeed similar to the first question in the text of chapter 2.3. The differential equation you need will represent the fact that the rate of change of the dye’s concentration is the rate of incoming dye minus the rate of outgoing dye. Let \( Q(t) \) represent the concentration of dye, in \( g/L \), at time \( t \).

In this case, the dye is incoming at a rate of 0, because the water is devoid of dye, and is outgoing at a rate that is equal to the concentration of the dye at any given time \( (Q(t)) \) times the amount of mixed water leaving the system (which is 2 \( L \) per minute). The key thing here is that because \( Q \) is the concentration, not the absolute number of grams of dye, the rate of departure of dye must also be measured as a proportion of the total liquid in the vat. So, when the rate of mixed water leaving the system is 2 \( L \) per minute, we have to think of that instead as 2/200, or 1\%, of the liquid per minute. So the differential equation becomes:

\[
Q' = 0 - \frac{2}{200} Q, \text{ or } Q' = -.01Q
\]

(The initial condition is that \( Q(0) \), the initial concentration, is 1 \( g/L \).)