1. Calculate the antiderivative. Be sure to give your final answer in terms of $x$.

$$
\int x^3 \sqrt{1 - x^2} \, dx
$$

Let $x = \sin \theta$; then $dx = \cos \theta \, d\theta$, so the integral becomes:

$$
\int \sin^3 \theta \sqrt{1 - \sin^2 \theta} \cos \theta \, d\theta = \int \sin^3 \theta \cos^2 \theta \cos \theta \, d\theta = \int \sin^3 \theta \cos^2 \theta \, d\theta
$$

This is a trigonometric integral, with a product of sine and cosine functions where one of the functions is raised to an odd power. Thus, the integral should be rewritten and solved using $u$-substitution.

$$
\int \sin^3 \theta \cos^2 \theta \, d\theta = \int \sin^2 \theta \cos^2 \theta \sin \theta \, d\theta = \int (1 - \cos^2 \theta) \cos^2 \theta \sin \theta \, d\theta
$$

Let $u = \cos \theta$; then $du = -\cos \theta \, d\theta$, so the integral becomes:

$$
- \int (1 - u^2)u^2 \, du = \int u^4 - u^2 \, du = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C
$$

$$
= \frac{1}{5} \cos^5 \theta - \frac{1}{3} \cos^3 \theta + C
$$

$$
= \frac{1}{5} \cos^5(\arcsin x) - \frac{1}{3} \cos^3(\arcsin x) + C
$$

Using a right triangle or the circular identities, we can recover the fact that $\cos(\arcsin x) = \sqrt{1 - x^2}$; thus, the final answer in terms of $x$ is:

$$
\frac{1}{5} (1 - x^2)^{5/2} - \frac{1}{3} (1 - x^2)^{3/2} + C
$$

2. Compute, using trigonometric substitution.

$$
\int_1^2 \frac{x}{\sqrt{x^2 - 1}} \, dx
$$

Note that this could have been done with a standard $u$-substitution, where $u = x^2 - 1$. That would be a good way to check your work on a problem like this. But if we proceed using trigonometric substitution, then we should use $x = \sec \theta$, in which case $dx = \sec \theta \tan \theta \, d\theta$. Using this substitution, in order to use the correct limits of integration, we must find a formula to recover $\theta$ from $x$, namely $\theta = \arccsc x$. This means that our limits of integration will become $\arcsin 1 = 0$, and $\arcsin 2 = \frac{\pi}{2}$.

$$
\int_0^{\pi/3} \sec \theta \sqrt{\sec^2 \theta - 1} \sec \theta \tan \theta \, d\theta = \int_0^{\pi/3} \sec^2 \theta \tan \theta \, d\theta = \int_0^{\pi/3} \sec^2 \theta \, d\theta
$$

$$
= \tan \theta \bigg|_0^{\pi/3} = \tan \frac{\pi}{3} - \tan(0) = \sqrt{3}
$$