Solutions

Show all work in the space provided or on scratch paper.

1. Integrate. \( \int \frac{t}{\sqrt{4-t^2}} \, dt \)
   
   \( u = 4 - t^2 \)
   \( du = -2t \, dt \)

   The integral becomes 
   
   \[-\frac{1}{2} \int u^{-1/2} \, du = -\frac{1}{2} \frac{u^{1/2}}{\frac{1}{2}} = -\sqrt{u} + C = -\sqrt{4-t^2} + C \]

2. Calculate the value of the definite integral. \( \int_{1}^{16} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \)
   
   \( u = \sqrt{x} \)
   \( du = \frac{1}{2\sqrt{x}} \, dx \)

   The integral becomes 
   
   \[\int_{1}^{4} e^{u} \, du \]
   
   \[= 2e^4 \bigg|_{1}^{4} = 2e^4 - 2e \]

3. Find the volume of the solid of revolution generated by revolving, about the line \( x = 1 \), the area enclosed by the functions \( y = x \) and \( y = x^2 + 3x - 3 \).

   The region is a parabola opening upward crossed by a line, and the intersection points can easily be found to be \(-3\) and \(1\). Because the region is to be revolved about a vertical axis, and the functions have independent variable \( x \), the easiest way to calculate the volume will be to use the method of cylindrical shells. The height of each shell will be the difference between the functions, and the radius will be the distance from \( x \) to the line \( x = 1 \). Thus \( r(x) = 1-x \) and \( h(x) = (x - (x^2 + 3x - 3)) = (-x^2 - 2x + 3) \).

   Using this data, the volume will be:
   
   \[ V = 2\pi \int_{-3}^{1} (1-x)(-x^2 - 2x + 3) \, dx = 2\pi \int_{-3}^{1} x^3 + x^2 - 5x + 3 \, dx \]
   
   \[= 2\pi \left( \frac{x^4}{4} + \frac{x^3}{3} - \frac{5x^2}{2} + 3x \right) \bigg|_{-3}^{1} = 2\pi \left( \frac{1}{4} + \frac{1}{3} - \frac{5}{2} + 3 - \frac{3^4}{4} + \frac{3^3}{3} + \frac{5 \cdot 3^2}{2} + 3 \cdot 3 \right) = \frac{64\pi}{3} \]