Calculus II - Midterm Exam Review, Spring 2013 [Shaw]

1. Calculate:

(a) \( \int_{-\frac{\pi}{2}}^{0} 3 \sin^2(x) \cdot \cos(x) \, dx \)

(b) \( \int \cos^3(\theta) \sin^2(\theta) \, d\theta \)

(c) \( \int_{1}^{e} \frac{\log_3(t)}{t} \, dt \)

(d) \( \int \left( e^x - \frac{5}{x} \right) \sqrt{e^x - 5 \ln(x)} \, dx \)

(e) \( \int \frac{\sqrt{t}}{t} \, dt \)

(f) \( \int \tan^3 \theta \sec^2 \theta \, d\theta \)

(g) \( \int_{0}^{\pi/2} t^2 \cos(t) \, dt \)

(h) \( \int \cos^2(5x) \, dx \)

(i) \( \int e^{2x} \sin(2x) \, dx \)

(j) \( \int \sec^5(\theta) \, d\theta \)

2. Find the area in the \( xy \)-plane enclosed between the curves \( y = 3x - 1 \) and \( y = x^2 + 3x - 2 \).

3. Use any appropriate method (e.g., discs, washers, or shells) to find the volume of each of the solids described below:

(a) The solid generated by revolving the curve \( f(x) = 1 + e^x \), \( 0 \leq x \leq \ln(2) \) about the \( x \)-axis.

(b) The solid generated by revolving the curve \( g(y) = \sqrt{y} \), \( 0 \leq y \leq 32 \) about the \( y \)-axis.

4. Find the length of the curve along \( x = y^4 + \frac{1}{8y^2} \) from \( y = 1 \) to \( y = 2 \).

5. Find the area of the surface of revolution obtained by revolving about the \( x \)-axis the curve \( h(x) = \sqrt{2x - x^2} \), \( 0 \leq x \leq 1 \).

6. A spring requires 0.2N of force to hold it steady when stretched to a distance of 0.01m from equilibrium. How much work is done by stretching the same spring from equilibrium to a distance of 0.1m?

7. Solve each of the separable differential equations; write your answers as functions in terms of \( x \).

(a) \( \frac{dy}{dx} = (-2x + 1)e^y \)

(b) \( \frac{1}{x^2 - x} \frac{dy}{dx} = \cos^2(y) \)

8. Determine whether \( f(t) = e^t + 5 \sin(t) \) is a solution to the differential equation \( y' + y'' = 2e^t \).

9. Solve the initial value problem \( (y + 1) \frac{dy}{dt} = yt \), \( y(0) = 1 \).

10. The mass of a particular kryptonite variety satisfies the differential equation \( P' = -600P \), where \( t \) is measured in years. What is the half-life of this kryptonite?