Solutions

You must show all work in order to receive full credit.

1. Integrate

\[ \int 7x e^{7x} \, dx \]

Let \( u = 7x \), and \( dv = e^{7x} \). Then \( du = 7 \, dx \) and \( v = \frac{1}{7} e^{7x} \), so the integral becomes:

\[
7x \cdot \frac{1}{7} e^{7x} - \int \frac{1}{7} \cdot e^{7x} \cdot 7 \, dx
\]

\[ = xe^{7x} - \int e^{7x} \, dx = xe^{7x} - \frac{1}{7} e^{7x} + C \]

2. Solve the differential equation. Write your answer as a function in terms of \( t \).

\[ \frac{dy}{dt} = t \sec(y) e^t \]

This is a separable differential equation, which means we must rewrite it in the form \( f(y) \frac{dy}{dt} = g(t) \):

\[
\frac{1}{\sec(y)} \frac{dy}{dt} = \frac{t}{e^t}
\]

However, each of these are written in such a way that their antiderivatives are not obvious. Thus we rewrite them:

\[
\cos(y) \frac{dy}{dt} = te^{-t}
\]

The left side is a straightforward antiderivative; the right requires integration by parts, with \( u = t \) and \( dv = e^{-t} \), just as above.

\[
\int \cos(y) \, dy = -te^{-t} + \int e^{-t} \, dt \\
\sin(y) = -te^{-t} - e^{-t} + C \\
y = \sin^{-1} (-te^{-t} - e^{-t} + C)
\]

3. Integrate

\[ \int_1^e x^3 \ln(x) \, dx \]

This is one iteration of integration by parts, with \( u = \ln(x) \), and \( dv = x^3 \). This gives us \( du = \frac{1}{x} \), and \( v = \frac{1}{4} x^4 \):

\[
\left. \frac{1}{4} x^4 \ln(x) \right|_1^e - \int_1^e \frac{1}{4} x^4 \cdot \frac{1}{x} \, dx = \left. \frac{1}{4} x^4 \ln(x) \right|_1^e - \int_1^e \frac{1}{4} x^3 \, dx = \left. \frac{1}{4} x^4 \ln(x) - \frac{1}{16} x^4 \right|_1^e \\
= \frac{1}{4} e^4 \ln(e) - \frac{1}{16} e^4 - \frac{1}{4} \ln(1) + \frac{1}{16} = \frac{3}{16} e^4 + \frac{1}{16}
\]