1. Let \( f(x) = x^2 \) and \( g(x) = \frac{1}{\sqrt{x-1}} \). Write down the functions \((f \circ g)(x)\) and \((g \circ f)(x)\), along with their domains.

\[
(f \circ g)(x) = \left( \frac{1}{\sqrt{x-1}} \right)^2
\]

The domain is \( \{ x : x > 1 \} = (1, +\infty) \): even though the function is squared, the fact that there is a square root in the definition means that \( x - 1 \) may not be negative.

\[
(g \circ f)(x) = \frac{1}{\sqrt{x^2 - 1}}
\]

The domain is \( \{ x : x < -1 \text{ or } x > 1 \} = (-\infty, -1) \cup (1, +\infty) \): \( x \) may not be between \(-1\) and \(1\) without the equation inheriting a square root of a negative number.

2. Let \( h(x) = e^{4x} - e \).

(a) Find (in simplest form) the value of \( h \left( \frac{1}{4} \right) \).

\[
h \left( \frac{1}{4} \right) = e^{4 \cdot \frac{1}{4}} - e = e - e = 0
\]

(b) Find the inverse function \( h^{-1}(x) \).

Recall the method to find the inverse function is to switch the variables, and then solve for \( y \):

\[
x = e^{4y} - e
\]

\[
x + e = e^{4y}
\]

Now you must take the natural logarithm of both sides in order to get the \( 4y \) out of the exponent:

\[
\ln(x + e) = \ln(e^{4y}) = 4y
\]

\[
y = \frac{1}{4} \ln(x + e), \text{ or, if you would prefer, } y = \ln(\sqrt[4]{x + e})
\]

3. Find the following:

I won’t include a long narration of how to find these values, as it would be repeating the long sections that already appear in the textbook, but the answers are below.

(a) \( \sin \left( \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} \)

(b) \( \cos^{-1} \left( \frac{\sqrt{2}}{2} \right) = \frac{\pi}{4} \)

(c) \( \tan \left( \cos^{-1}(0.3) \right) = \frac{\sqrt{0.91}}{0.3} \)