Solutions

1. Find \( \frac{d}{dx}[x \sin(x)] \)
   
   This one uses just the product rule:
   
   \( 1 \cdot \sin(x) + x \cos(x), \text{ or } \sin(x) + x \cos(x) \)

2. Find \( \frac{d}{dt}\bigg|_{t=0} \left[ 13 \sqrt[4]{e^t + 7t^2 + 1} \right] \)
   
   Recall that \( \sqrt[4]{A} = A^{\frac{1}{4}}, \) which means we want the derivative at zero of the function:
   
   \( 13 \cdot (e^t + 7t^2 + 1)^{\frac{3}{4}} \)

   Use the chain rule: the inside function is \( e^t + 7t^2 + 1, \) and the outside function is a power rule.
   
   \[ \frac{13}{4} \left( e^t + 7t^2 + 1 \right)^{-\frac{3}{4}} (e^t + 14t) \]

   Now, at \( x = 0, \) this becomes:
   
   \[ \frac{13}{4} \cdot (1 + 0 + 1)^{-\frac{3}{4}} (1 + 0) = \frac{13}{4} \cdot 2^{-\frac{3}{4}} \]

3. Find \( \frac{d}{dt} \left[ \cos^2(t) \right] \)
   
   This one requires the quotient rule; as a separate calculation, note that the derivative of \( \cos^2(t) \) requires the chain rule, with inside function \( \cos(t) \) and outside function \( t^2. \)
   
   \[ \frac{(-2 \cos(x) \sin(x))(x^3 - 2) - 3x^2 \cos^2(x)}{(x^3 - 2)^2} \]

BONUS. Find \( \frac{dy}{dx} \) for the expression.

\[ \frac{y}{x} - x^2 = \cos(xy) \]

Recall that for implicit differentiation, you must treat \( y \) as a function unto itself whose derivative is \( \frac{dy}{dx}, \) and as such the chain rule is required any time \( y \) makes an appearance. All other rules, including the product and quotient rules, still apply.

\[ \frac{dy}{dx} \cdot \frac{x - y \cdot 1}{x^2} - 2x = - \left( 1 \cdot y + x \cdot \frac{dy}{dx} \right) \cdot \sin(xy) \]

Collect the \( \frac{dy}{dx} \) terms to get

\[ \frac{dy}{dx} = \frac{-y \sin(xy) + 2x + \frac{y}{x}}{1 + x \sin(y)} \]