1. (a) The rocket's airtime corresponds to the time until its height is zero, or when $s(t) = 0$. So solve:

$$0 = s(t) = 80t - 16t^2 = t(80 - 16t)$$

$$t = 0 \text{ and } t = \frac{80}{16} = 5$$

Thus the rocket lands after 5 seconds.

(b) This now means we have to find $v(5)$, recalling that $v(t) = s'(t)$:

$$s'(t) = 80 - 32t; \quad s'(5) = 80 - 32 \cdot 5 = 80 - 160 = -80$$

So, the rocket's velocity is -80 feet per second, or 80 feet per second downward.

2. (a) To find this limit, we need to work with a common denominator, which in this case means $5 \cdot (5 + h)$.

$$\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{\frac{5\cdot 1}{5\cdot(5+h)} - \frac{1\cdot(5+h)}{5\cdot(5+h)}}{h} = \lim_{h\to 0} \frac{\frac{5}{5\cdot(5+h)} - \frac{(5+h)}{5\cdot(5+h)}}{h} = \lim_{h\to 0} \frac{\frac{5-5-h}{5\cdot(5+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{5\cdot(5+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-1}{5 \cdot (5 + h)} = \frac{-1}{25}$$

(b) This limit is of a rational function, so we should try to factor and look for common factors that may cancel each other out:

$$\lim_{x\to 2} \frac{2x^2 - 8}{2 - x} = \lim_{x\to 2} \frac{2(x^2 - 4)}{2 - x} = \lim_{x\to 2} \frac{2(x + 2)(x - 2)}{-(x - 2)} = \lim_{x\to 2} \frac{2(x + 2)}{-1} = -8$$

3. To find the equation of the tangent line, you are given the point $(0, 1)$, so we first find the slope (namely the value of the derivative at the x-value), and then use that along with the point-slope formula to find the equation:

$$f'(x) = 2e^{2x} + 2$$
$$f'(0) = 2 + 2 = 4$$

$$y - 1 = 4 \cdot (x - 0)$$
$$y = 4x + 1$$

4. (a) $g(t) = \pi \sin^{-1}(t)$

$g'(t) = \dfrac{\pi}{\sqrt{1 - x^2}}$

(b) $y = 4x^\pi - x$

$\dfrac{dy}{dx} = 4\pi x^{\pi-1} - 1$

(c) $h(x) = \dfrac{1}{x} + \dfrac{2}{x^2}$

$h'(x) = -x^{-2} - 4x^{-3}$

5. (a) $\dfrac{d}{dx}\left[(e^x + x)\sin(2x)\right]$ [product rule]

$$(e^x + 1)\,(\sin(2x)) + (e^x + x)\,(2\cos(2x))$$

(b) $\dfrac{d}{dt}\left[\dfrac{t^2 - 2t + 1}{t^7 + t^4 + 1}\right]$ [quotient rule]

$$\frac{(2t - 2)\left(t^7 + t^4 + 1\right) - \left(t^2 - 2t + 1\right)\left(7t^6 + 4t^3\right)}{\left(t^7 + t^4 + 1\right)^2}$$

(c) $\dfrac{d^2}{dx^2}[\sin(x)\cos(x)]$ [product rule, twice]

$$\frac{dy}{dx} = \cos^2(x) - \sin^2(x)$$

$$\frac{d^2y}{dx^2} = -2\cos(x)\sin(x) - 2\sin(x)\cos(x) = -4\sin(x)\cos(x)$$

6. Using implicit differentiation:

$$ye^y - x = 1 - x^2$$

$$\frac{dy}{dx}e^y + y\frac{dy}{dx}e^y - 1 = -2x$$

$$\frac{dy}{dx} = \frac{-2x+1}{e^y(y+1)}$$

7. (a) Domain of $f(x) = \mathbb{R}$. Range of $f(x) = [-4, \infty)$.
   Domain of $g(x) = [-8, \infty)$. Range of $g(x) = [0, \infty)$.

   (b) $(g \circ f)(x) = \sqrt{x^2 - 4 + 8} = \sqrt{x^2 + 4}$, so the domain is $\mathbb{R}$, and the range is $[2, \infty)$.

8. (a) $\lim_{x\to 0} f(x) = 2$ (b) $\lim_{x\to 2^-} f(x) = 1$ (c) $\lim_{x\to -2} f(x)$ DNE

9. (a) $\dfrac{d}{dx}\left[\cos\left(x^4 - e^x\right)\right]$ [Chain rule]

   $$\frac{dy}{dx} = \left(4x^3 - e^x\right)\left(-\sin\left(x^4 - e^x\right)\right)$$

   (b) Find $\dfrac{d}{dt}\left[\ln\left(\cos^2(t)\right)\right]$ [Chain rule]

   $$\frac{dy}{dt} = \frac{-2\cos(t)\sin(t)}{\cos^2(t)} = -2\frac{\sin(t)}{\cos(t)} = -2\tan(t)$$

10. Any parabola facing up with a vertex at x=1 would do here. (Try looking at the graph of $y = x^2 - 2x + 1$, for instance.)

BONUS If $f(x)$ and $f'(x)$ both pass through $(\pi, 2)$, then $f(\pi) = 2$ and $f'(\pi) = 2$. Thus the equation of the tangent line is gotten by:

$$y - 2 = 2(x - \pi)$$

$$y = 2x - 2\pi + 2$$