1. Find the function $g(x)$ that satisfies the initial value problem below:

$$g'(x) = \frac{2 \ln(x)}{x} \quad g(1) = 13$$

$$g(x) = \int \frac{2}{\ln(x)} \, dx$$

Let $u = \ln(x)$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\int 2u \, du = \int 2 \frac{u^2}{2} + C$$

$$g(x) = (\ln(x))^2 + C$$

$$g(1) = (\ln(1))^2 + C = 13$$

$$0 + C = 13$$

$$C = 13$$

$$g(x) = (\ln(x))^2 + 13$$

2. Find $\int_0^\frac{\pi}{2} \cos(x) e^{\sin(x)} \, dx$

Let $u = \sin(x)$

$$\frac{du}{dx} = \cos(x)$$

$$\int_{\sin(0)}^{\sin(\frac{\pi}{2})} du = \int_0^1 e^u \, du$$

$$= e^u \bigg|_0^1 = e - e^0 = e - 1$$

Bonus 1 Estimate the area below the curve $y = \sqrt{x - 1}$ and above the x-axis on the interval [3, 4], using 5 subdivisions and the midpoints of the rectangles.

The length of the interval is 1, and given that there are 5 subdivisions, the width of each rectangle $\delta x$ is $\frac{1}{5} = 0.2$. Thus the midpoints of the rectangles are 3.1, 3.3, 3.5, 3.7, and 3.9. Then the area estimate becomes:

$$0.2(f(3.1) + f(3.3) + f(3.5) + f(3.7) + f(3.9))$$

$$= 0.2(\sqrt{3.1 - 1} + \sqrt{3.3 - 1} + \sqrt{3.5 - 1} + \sqrt{3.7 - 1} + \sqrt{3.9 - 1})$$

$$= 0.2(\sqrt{2.1} + \sqrt{2.3} + \sqrt{2.5} + \sqrt{2.7} + \sqrt{2.9})$$

which, FYI, is approximately 1.58.

Bonus 2 Find the area enclosed by the graphs of $x = 0, x = 1, y = x - 1$, and $y = e^{-x}$

The graph of the region is to the right, in which it is clear that the top function is $y = e^{-x}$. Thus, the integral becomes:

$$\int_0^1 e^{-x} - (x - 1) \, dx = -e^{-x} - \frac{x^2}{2} + x \bigg|_0^1$$

$$= -\frac{1}{e} - \frac{1}{2} + 1 + \frac{3}{2} - \frac{1}{e}$$