Calculus I - Sample Exam and Review for Midterm, 2010 [Shaw]

1. A rock is thrown straight up into the air allowed to fall to the ground. After t seconds, the rock's height above the ground, in feet, is given by the expression

$$s(t) = 7 + 10t - 16t^2$$

   (a) Write a general formula for the rock's velocity at time t.

   (b) How many seconds does the rock spend traveling upward?

2. (a) Find the limit: $\lim_{x \to 7} \dfrac{2x^2 - 14x}{-x + 7}$

   (b) Find the limit: $\lim_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

3. Find the equation for the tangent line to the function $f(x)$ at the point $(\frac{\pi}{2}, 0)$.

$$f(x) = \cos^2(x) + 2x - \pi$$

4. Find the derivative of each function below.

   (a) $y = 3x^3 - 2x^2 + 1$

   (b) $y = 2 - \sqrt[3]{x} + \frac{\pi}{x}$

   (c) $y = 3e^x + 1$

5. (a) Find $\dfrac{d^2}{dx^2}\left[xe^{3x}\right]$

   (b) Find $\dfrac{d}{dx}\left[\dfrac{6x - 7}{x^4 - x^2 + 1}\right]$

   (c) Find $\dfrac{d}{dt}\left[(\tan(x)) \cdot \left(x + x^3\right)\right]$

6. Find $\frac{dy}{dx}$, for the relation below.

$$\left(y^3 - y\right) x = e^y$$

7. Upon its initial start, a car travels forward so that at time t, in seconds, the car has traveled $s(t)$ feet, where $s(t) = 8.5t^2$. What is the acceleration of the car after 2.3 seconds?

8. Consider the function

$$g(x) = \begin{cases} 4x - 3 & x \leq 3 \\ 2 & 3 < x \leq 4 \\ \sqrt{x} & x > 4 \end{cases}$$

   (a) Sketch the graph of $g(x)$.

   (b) Find $\lim_{x \to 3} g(x)$.

   (c) Find $\lim_{x \to 4} g(x)$.

9. (a) Find $f'(x)$, if $f(x) = \sin^{-1}(3x)$.

   (b) Find $g'(1)$, when $g(x) = 1 + \tan^{-1}(x)$.

10. (a) Find $\dfrac{d}{dx}\left[\ln\left(x^3 + 4\cos(x)\right)\right]$

    (b) If $y = 3u + 7$ and $u = 6t^3 - e^t$, find $\dfrac{dy}{dt}$.

11. Find the equation for all horizontal and vertical asymptotes of the function $h(x) = \dfrac{x^2 - 3x + 1}{x^3 - 1}$.