1. The linearization near \( x_0 = 25 \) is \( L(x) = f(25) + f'(25) \cdot (x - 25) \). So we need \( f(25) \), which is \( \sqrt{25} = 5 \), and \( f'(25) \).

\[
f'(x) = \frac{1}{2} x^{-\frac{1}{2}}, \quad \text{so } f'(25) = \frac{1}{2} \cdot \sqrt{25} = \frac{1}{10}
\]

Thus \( L(x) = 5 + \frac{1}{10} (x - 25) = 5 + \frac{1}{10} x - \frac{25}{10} = \frac{1}{10} x + \frac{5}{2} \).

Then the estimate will be \( L(24.9) = 2.49 + 2.5 = 4.99 \).

2. To find the absolute extreme points on a closed interval, we find all critical points and evaluate the function at these points and at the endpoints. The largest value will be the absolute maximum and the smallest value will be the absolute minimum.

\( g'(x) = 3x^2 - 4x + 1 \). This function exists everywhere so the only critical points will be where \( g'(x) = 0 \). The function factors to \( g'(x) = (3x - 1)(x - 1) \), which is zero at \( x = 1 \) and \( x = \frac{1}{3} \).

Now we check each of those values:

\[
\begin{align*}
g(0) &= 1; \quad g(10) = 811; \quad g(1) = 1; \quad g\left( \frac{1}{3} \right) = \frac{31}{27}
\end{align*}
\]

So the absolute maximum occurs at \((10, 811)\), and the absolute minimum is achieved at \((1, 1)\) or \((0, 1)\) [either is acceptable].

3. We set \( x \) to be the distance the player travels down the field, and \( y \) to be the distance from the runner to the observer. A simplified labeled picture is below.

\[
\begin{align*}
&\text{The given information in the problem states that } \frac{dx}{dt} \text{ is constant at 9 yps. We want to know what } \frac{dy}{dt} \text{ is at the time when } x = 25. \\
\text{The equation relating all of these values is the Pythagorean theorem, namely that } x^2 + 30^2 = y^2. \text{ We solve this equation for } y \text{ and then differentiate.}
\end{align*}
\]

\[
\begin{align*}
y &= \sqrt{x^2 + 900} \\
\frac{dy}{dt} &= \frac{1}{2} (x^2 + 900)^{-\frac{1}{2}} \cdot 2x \frac{dx}{dt} = \frac{2x \frac{dx}{dt}}{2\sqrt{x^2 + 900}} = \frac{x \frac{dx}{dt}}{\sqrt{x^2 + 900}}
\end{align*}
\]

Substituting \( x = 25 \) and \( \frac{dx}{dt} = 9 \), we get:

\[
\frac{dy}{dt} = \frac{25 \cdot 9}{\sqrt{25^2 + 900}} = \frac{45}{\sqrt{61}} \approx 5.76 \text{ yps. } \text{[Any correct form would have been fine.]}\]

BONUS

- Prove there is a root. This is because \( h(-2) = 16 + 8 - 4 = 20 > 0 \) and \( h(-1) = 1 + 1 - 4 = -2 < 0 \), so by the Intermediate Value Theorem the function crosses the \( x \)-axis between \(-2 \) and \(-1\).

- Prove there is at most one root. This is the part that uses Rolle's theorem: if there is more than one root, then there would be a place in the interval \((-2, -1)\) where the derivative is zero. But \( h'(x) = 4x^3 - 3x^2 = x^2(4x - 3) \), which is zero only at \( x = 0 \) and \( x = \frac{3}{4} \). Thus the derivative is nonzero in the prescribed interval, and as there is no second root.