Name ____ **Calculus I - Final Exam review, Fall 2010 [Shaw]**

1. Find the derivative:

   (a) $f(z) = 3z^7 - \sqrt{z}$ (b) $g(x) = \sin(x^3 - x^2 - x - 1)$ (c) $h(t) = t \cdot e^{\sin(t)}$

2. Use implicit differentiation to find $\left.\dfrac{dy}{dx}\right|_{(0,0)}$ for the function defined below:

$$2x^2 - 3y + y^2 = \cos(x) - 1$$

3. Find the equation for the tangent line to the below equation at the point where $x = e$:

$$y = (\ln x)^5$$

4. Consider the function $f(x) = x \sin x + \cos x$, with domain $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$.
   (a) Find the value of all critical points for $f(x)$ on the given domain.
   (b) For each of these critical points, use the second derivative test to determine whether it is a local maximum, a local minimum, or neither.
   (c) Find the x- and y-value of the absolute minimum and absolute maximum for f on the given domain.

5. A 15-foot tall ladder is leaning against the wall of a building, and sliding down the wall. Suppose that when the base of the ladder is 9 feet from the base of the wall, it is moving away from the wall at a rate of 1 foot per second. How fast is the top of the ladder sliding downward at the same point in time?

6. Find numbers x and y such that $x^2 y = 4000$ and $x + y$ is as small as possible.

7. Find the following:

   (a) $\displaystyle\int_{-1}^{1} \cos(\pi x)\, dx$ (b) $\displaystyle\int \frac{\sec^2(\ln(x))}{x}\, dx$ (c) $\displaystyle\int_0^1 \left(3x^2 + 1\right) e^{x^3 + x}\, dx$

8. Use the linearization formula $L(x) = f(x_0) + f'(x_0)(x - a)$ to estimate the value of $\sqrt[3]{26.9}$.

9. Find the function $f(x)$ which satisfies $f'(x) = x^2 - x + 1$ and $f(1) = 1$.

10. A particle is measured traveling back and forth on a straight line for 5 nanoseconds. At time t nanoseconds, the particle's location relative to the starting point in millimeters is $s(t) = t^3 - t^2 + \frac{1}{10-t}$.
    (a) Find an equation for the particle's velocity during the 5 nanosecond measurement period.
    (b) What is the particle's acceleration at time $t = 1$?

11. Find the limit: $\displaystyle\lim_{\theta \to \pi} \sin(\theta) - \frac{\pi}{\theta}$

12. Find the equation for all horizontal and vertical asymptotes of the function. For each type of asymptote, if there are none, state "none."

$$f(x) = \frac{x^4 + 2}{x^5 - 1}$$

13. Sketch the graph of the function $y = x^3 - 3x$, including all intercepts, local maximum and minimum points, intervals of increasing/decreasing and concavity, and inflection points.

14. Estimate the area under the curve $y = \sqrt{x}$ on the interval $[0, 4]$, using $n = 4$ rectangles, and right endpoints. You do not need to simplify your answer.

15. Given the function $g(x) = 1 - e^{4x}$, find the inverse function $g^{-1}(x)$.