1. (a) Remember that Newton-Raphson is only used to estimate which values of \( x \) make a certain function equal to zero. So, to find when \( x = \sqrt{10} \), we need to use the function \( f(x) = x^2 - 10 \), which is zero when \( x = 10 \). Note also that \( f'(x) = 2x \). We are told \( x_0 = 3 \), so we calculate:

\[
x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{3^2 - 10}{(2)(3)} = 3 - \frac{1}{6} = \frac{19}{6}
\]

\[
x_2 = \frac{19}{6} - \frac{(\frac{19}{6})^2 - 10}{2 \cdot (\frac{19}{6})}
\]

And we stop there, since \( n = 2 \), and the problem says not to simplify.

(b) i. To use the formula for calculating a Taylor polynomial about \( x = a \), we need to know the derivatives \( \sin(x) \) for the value \( x = \frac{\pi}{2} \).

\[
f(x) = \sin(x) \quad f'(x) = \cos(x) \quad f''(x) = -\sin(x)
\]

\[
f \left( \frac{\pi}{2} \right) = 1 \quad f' \left( \frac{\pi}{2} \right) = 0 \quad f'' \left( \frac{\pi}{2} \right) = -1
\]

So, the Taylor polynomial is:

\[
p_2(x) = 1 + 0 \cdot \left( x - \frac{\pi}{2} \right) + \frac{-1}{2!} \cdot (x - \frac{\pi}{2})^2 = 1 - \frac{1}{2} \left( x - \frac{\pi}{2} \right)^2
\]

ii. Thus, \( p_2 \left( \frac{11\pi}{20} \right) = 1 - \frac{1}{2} \left( \frac{11\pi}{20} - \frac{\pi}{2} \right)^2 = 1 - \frac{1}{2} \left( \frac{\pi}{20} \right)^2 = \frac{800 - \pi^2}{800} \)

2. (a) \( 1.23 = 1 + \left( \frac{23}{100} + \frac{23}{100^2} + \frac{23}{100^3} + \cdots \right) \)

The part in the parentheses is a geometric series with \( a = \frac{23}{100} \), and \( r = \frac{1}{100} \). Thus the whole answer is \( 1 + \frac{23}{100} = 1 + \frac{23}{99} = 1 + \frac{23}{99} \) or \( \frac{122}{99} \).

(b) \( \sum_{n=0}^{\infty} \frac{2^n}{7^n} \) is a geometric series, which in this case is easy to interpret: if we write out the first few terms, we get \( 2 + 2 \cdot \frac{2}{7} + 2 \cdot \left( \frac{2}{7} \right)^2 + \cdots \). We can find that \( a = 2 \) and \( r = \frac{2}{7} \), so the sum is \( \frac{2}{1 - \frac{2}{7}} = \frac{2}{\frac{5}{7}} = \frac{14}{5} \).

3. \( \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \left( \frac{1}{n} \right) \) is not a geometric series, so we need to use the integral test, and find out whether or not \( \int_1^{\infty} \frac{1}{x^2} \sin \left( \frac{1}{x} \right) \) is convergent. This integral is a \( u \)-substitution with \( u = \frac{1}{x} \).

It turns out to be convergent, with value equal to:

\[
\lim_{t \to \infty} \cos \left( \frac{1}{t} \right) - \cos(1) = \cos(0) - \cos(1)
\]

Even though we don’t know what \( \cos(1) \) is, we know this means the series is convergent.
4. (a) This one involves the Taylor series for \( \frac{1}{1 - x} \). The function \( \frac{x^2}{1 + x^3} \) can be rewritten as 
\[
x^2 \cdot \frac{1}{1 - (-x^3)}
\]
so we have a substitution and a multiplication problem:
\[
x^2 \cdot \frac{1}{1 - (-x^3)} = x^2 \left( 1 + (-x^3) + (-x^3)^2 + (-x^3)^3 + (-x^3)^4 + \cdots \right)
\]
\[= x^2 - x^5 + x^8 - x^{11} + x^{14} - \cdots\]

(b) If \( g(x) = 1 - \frac{1}{2!} x^2 + \frac{1}{3!} x^4 - \frac{1}{4!} x^6 + \frac{1}{5!} x^8 - \cdots \), then
\[
\int 2g(x) \, dx = 2 \left( x - \frac{1}{3 \cdot 2!} x^3 + \frac{1}{5 \cdot 3!} x^5 - \frac{1}{7 \cdot 4!} x^7 + \frac{1}{9 \cdot 5!} x^9 - \cdots \right) + C.
\]
Setting \( G(0) = 2 \) gives us \( C = 2 \), so
\[
G(x) = 2 + 2x - \frac{2}{3 \cdot 2!} x^3 + \frac{2}{5 \cdot 3!} x^5 - \frac{2}{7 \cdot 4!} x^7 + \frac{2}{9 \cdot 5!} x^9 - \cdots.
\]

5. (a) \( E(x) = 0 \cdot \frac{12}{30} + 1 \cdot \frac{11}{30} + 2 \cdot \frac{5}{30} + 3 \cdot \frac{2}{30} = \frac{27}{30} \)

(b) \( V(x) = \frac{12}{30} \left( 0 - \frac{27}{30} \right)^2 + \frac{11}{30} \left( 1 - \frac{27}{30} \right)^2 + \frac{5}{30} \left( 2 - \frac{27}{30} \right)^2 + \frac{2}{30} \left( 3 - \frac{27}{30} \right)^2 \)

(c) Finally, the standard deviation \( \sigma(x) = \sqrt{V(x)} \).