1. Find the integrals indicated below:

(a) \[ \int 3e^{-4x+13} \, dx = \frac{3}{-4}e^{-4x+13} + C \]

(b) \[ \int z^3 + \frac{1}{2z^3} \, dz = \int z^3 + \frac{1}{2} \cdot \frac{1}{z^2} \, dz = \frac{1}{4} \left(z^4 - \frac{1}{z^2}\right) \]

(c) \[ \int_{-3}^{-1} \frac{3}{t} \, dt = -3 \ln |t| \bigg|_{-3}^{-1} = -3(\ln |-3| - \ln |-4|) = -3(\ln(3) - \ln(4)) = -3\ln \left(\frac{3}{4}\right) = \ln \left(\frac{64}{27}\right) \]

2. (a) Using left endpoints with \( n = 4 \), estimate the area between the curve \( f(x) = 4 - x^2 \) and the \( x \)-axis.

If you draw the graph, you see that this is an inverted parabola, and the intersection points with the \( x \)-axis are found by setting \( 4 - x^2 = 0 \), which gives \( x = -2 \) and \( x = 2 \). So the area will be from \( a = -2 \) to \( b = 2 \).

\( \Delta x = \frac{2 - (-2)}{4} = 1 \), so the left endpoints are \(-2, -1, 0, 1\), and the approximate area is:

\[ A \approx 1(f(-2) + f(-1) + f(0) + f(1)) = 4 - (-2)^2 + 4 - (-1)^2 + 4 - (0)^2 + 4 - (1)^2 = 0 + 3 + 4 + 3 = 10 \]

(b) Find the function \( f(x) \) which has the following properties:

\[ f''(x) = 4e^{2x} \]
\[ f'(1) = 2e^2 \]
\[ f(1) = e^2 + 3 \]

We need to find \( f'(x) \) first, using the antiderivative of \( f''(x) \) and the information in the problem to solve for \( C \), and then do the same thing again to get \( f(x) \) from \( f'(x) \):

\[ f'(x) = \int 4e^{2x} \, dx = \frac{4}{2}e^{2x} + C_0 = 2e^{2x} + C_0 \]
\[ f'(1) = 2e^{2\cdot1} + C_0 = 2e^2, \text{ so } C_0 = 0 \text{ and } f'(x) = 2e^{2x} \]

\[ f(x) = \int 2e^{2x} \, dx = \frac{2}{2}e^{2x} + C_1 = e^{2x} + C_1 \]
\[ f(1) = e^{2\cdot1} + C_1 = e^2 + 3, \text{ so } C_1 = 3 \text{ and } f(x) = e^{2x} + 3 \]

(c) Determine whether or not it is true that: \( \int \ln(x) \, dx = x \ln(x) - x + C \)

Check by taking the derivative of the right side of the equation:

\[ \frac{d}{dx} [x \ln(x) - x + C] = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x), \text{ so YES.} \]

3. (a) Find the area of the region enclosed by the functions \( y = \sqrt{-x} \) and \( y = -\frac{1}{3}x \) (your answer should be simplified).

The intersection points are found by setting the functions equal:

\[ \sqrt{-x} = -\frac{1}{3}x. \text{ Square both sides to obtain:} \]

\[ -x = \frac{1}{9}x^2, \text{ which is a quadratic.} \]

\[ \frac{1}{9}x^2 + x = 0 \]
\[ x \left( \frac{1}{9} x + 1 \right) = 0 \]
\[ x = 0, \ x = -9 \]

Now, either by graphing the functions or checking a value in between 0 and -9 (-1 works), we can see that \( \sqrt{-1} \) is on top. So the area is the integral of the difference of the functions:

\[ A = \int_{-9}^{0} \sqrt{-x + 1} - \frac{1}{3} x \]

When taking this integral, be very careful about what happens to the negative sign inside the square root:

\[ = -\frac{2}{3} (-x)^{3/2} + \frac{1}{6} x^2 \bigg|_{-9}^{0} = 0 - \left( -\frac{2}{3} (9)^{3/2} + \frac{1}{6} (-9)^2 \right) = \frac{2}{3} \cdot 27 - \frac{81}{6} = \frac{9}{2} \]

(b) Suppose that the function \( F(x) \) has the property that \( F(-6) = 3 \) and \( F(3) = 10 \). The graph of its derivative \( F'(x) \) is pictured below. What is the area of the region \( A \)?

This is just by the definitions! The area is:

\[ \int_{-6}^{3} F'(x) \ dx = F(x) \bigg|_{-6}^{3} = F(3) - F(-6) = 10 - 3 = 7 \]

4. (a) Suppose money is deposited daily into a bank account over a period of 10 years. The account has interest rate 4\%, and the deposits add up to $100 per month. Use the future value of a continuous income stream formula (below) to estimate how much money will be in the bank account at the end of the 10 years.

\[ A = \int_{0}^{N} Ke^{r(N-t)} \ dt \]

The challenge here is to pick out each of the values to go into the equation. It is clear that \( r = 0.04 \) and \( N = 10 \), but notice that since the deposits are about $100 per month, the annual deposits (which is what \( K \) is) are $1200, so \( K = 1200 \). Then you set up and solve the integral:

\[ A = \int_{0}^{10} 1200e^{0.04 \cdot 0.8} \]

\[ = 1200 \cdot 0.04 e^{0.04 \cdot 0.8} \bigg|_{0}^{10} = -30000(e^{0} - e^{4}) = 30000(e^{4} - 1) \]

(b) On a small racetrack, a car takes a rolling start and drives from point A to point B in 8 seconds. The velocity at time \( t \) seconds is given by the equation \( v(t) = 12t + 20 \) (in m/sec).

i. What is the average velocity of the car over its 8 seconds of travel?

\[ \frac{1}{8} \int_{0}^{8} (12t + 20) \ dt, \text{ which I trust you to integrate.} \]

ii. What is the total distance covered from point A to point B?

\[ \int_{0}^{8} (12t + 20) \ dt, \text{ which I trust you to integrate.} \]

(c) Set up but do not solve an integral to determine the volume of the solid of revolution obtained by rotating the function \( f(x) = (\ln(x))^5 \) around the x-axis from \( x = 1 \) to \( x = 5 \).

\[ \int_{1}^{5} \pi (\ln(x))^10 \ dx \]
(d) Determine the consumer surplus for the demand function \( f(x) = \sqrt{50 - x} + 145 \) at sales level \( A = 25 \). Use the graph pictured below to help you.

\[ A = 25, \ B = f(25) = 150, \] so the integral to get the surplus is:

\[
\int_0^{25} (\sqrt{50 - x} + 145 - 150) \, dx,
\]

which I trust you to simplify and then integrate.

5. (a) Let \( f(x, y, z) = xy + xyz + xe^z \).

i. What is \( f(2, 3, 0) \)?

\[ f(2, 3, 0) = 2 \cdot 3 + 0 + 2e^0 = 6 + 2 = 8 \]

ii. Find \( \frac{\partial f}{\partial x} \), \( \frac{\partial f}{\partial y} \), and \( \frac{\partial f}{\partial z} \).

\[
\frac{\partial f}{\partial x} = y + yz + e^z; \quad \frac{\partial f}{\partial y} = x + xz; \quad \frac{\partial f}{\partial z} = xy + xe^z
\]

(b) Draw the level curves at heights \( z = 0, \ z = 1, \ z = 2 \), for the function \( g(x, y) = 2x - y \).

The level curves are \( y = 2x, \ y = 2x - 1, \ and \ y = 2x - 2, \) which are easy to draw (all straight lines with slope 2 translated down by 1 each time). Drawing them together on the same axes is fine, as is drawing three separate graphs.