1. *Find the integrals indicated below:*

   (a) $\int 3e^{-4x+13}\,dx$

   $= \frac{3}{-4}e^{-4x+13} + C$

   (b) $\int z^3 + \frac{1}{2z^3}\,dz$

   $= \int z^3 + \frac{1}{2}\cdot z^{-3}\,dz = \frac{z^4}{4} + \frac{1}{-2}\cdot\frac{1}{2}z^{-2} = \frac{1}{4}\left(z^4 - \frac{1}{z^2}\right)$

   (c) $\int_{-4}^{-3} -\frac{3}{t}\,dt = -3\ln|t|\Big|_{-4}^{-3}$

   $= -3(\ln|-3| - \ln|-4|) = -3(\ln(3) - \ln(4)) = -3\ln\left(\frac{3}{4}\right) = \ln\left(\left(\frac{3}{4}\right)^{-3}\right) = \ln\left(\frac{64}{27}\right)$

2. (a) *Using left endpoints with $n = 4$, estimate the area between the curve $f(x) = 4 - x^2$ and the x-axis.*
   If you draw the graph, you see that this is an inverted parabola, and the intersection points with the x-axis are found by setting $4 - x^2 = 0$, which gives $x = -2$ and $x = 2$. So the area will be from $a = -2$ to $b = 2$.
   $\Delta x = \frac{2-(-2)}{4} = 1$, so the left endpoints are -2, -1, 0, 1, and the approximate area is:

   $$A \approx 1(f(-2)+f(-1)+f(0)+f(1)) = 4-(-2)^2+4-(-1)^2+4-(0)^2+4-(1)^2 = 0+3+4+3 = 10$$

   (b) *Find the function $f(x)$ which has the following properties:*

   $$f''(x) = 4e^{2x}$$
   $$f'(1) = 2e^2$$
   $$f(1) = e^2 + 3$$

   We need to find $f'(x)$ first, using the antiderivative of $f''(x)$ and the information in the problem to solve for C, and then do the same thing again to get $f(x)$ from $f'(x)$:

   $$f'(x) = \int 4e^{2x}\,dx = \frac{4}{2}e^{2x} + C_0 = 2e^{2x} + C_0$$

   $$f'(1) = 2e^{2\cdot 1} + C_0 = 2e^2, \text{ so } C_0 = 0 \text{ and } f'(x) = 2e^{2x}$$

   $$f(x) = \int 2e^{2x}\,dx = \frac{2}{2}e^{2x} + C_1 = e^{2x} + C_1$$

   $$f(1) = e^{2\cdot 1} + C_1 = e^2 + 3, \text{ so } C_1 = 3 \text{ and } f(x) = e^{2x} + 3$$

   (c) *Determine whether or not it is true that:* $\int \ln(x)\,dx = x\ln(x) - x + C$
   Check by taking the derivative of the right side of the equation:

   $$\frac{d}{dx}\left[x\ln(x) - x + C\right] = 1\cdot\ln(x) + x\cdot\frac{1}{x} - 1 = \ln(x), \text{ so YES.}$$

3. (a) *Find the area of the region enclosed by the functions $y = \sqrt{-x}$ and $y = -\frac{1}{3}x$ (your answer should be simplified).*
   The intersection points are found by setting the functions equal:

   $$\sqrt{-x} = -\frac{1}{3}x. \text{ Square both sides to obtain:}$$

   $$-x = \frac{1}{9}x^2, \text{ which is a quadratic.}$$

   $$\frac{1}{9}x^2 + x = 0$$

$$x\left(\frac{1}{9}x+1\right)=0$$

$$x=0,\ x=-9$$

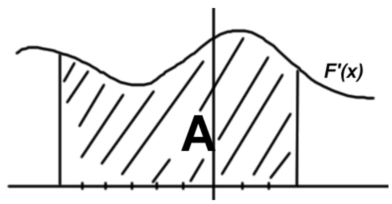
Now, either by graphing the functions or checking a value in between 0 and -9 (-1 works), we can see that $\sqrt{-1}$ is on top. So the area is the integral of the difference of the functions:

$$A=\int_{-9}^{0}\sqrt{-x}+\frac{1}{3}x$$

When taking this integral, be very careful about what happens to the negative sign inside the square root:

$$=-\frac{2}{3}(-x)^{3/2}+\frac{1}{6}x^2\Big|_{-9}^{0}=0-\left(-\frac{2}{3}(9)^{3/2}+\frac{1}{6}(-9)^2\right)=\frac{2}{3}\cdot 27-\frac{81}{6}=\frac{9}{2}$$

(b) *Suppose that the function $F(x)$ has the property that $F(-6)=3$ and $F(3)=10$. The graph of its derivative $F'(x)$ is pictured below. What is the area of the region A?*



This is just by the definitions! The area is:

$$\int_{-6}^{3}F'(x)\,dx=F(x)\Big|_{-6}^{3}=F(3)-F(-6)=10-3=7$$

4. (a) *Suppose money is deposited daily into a bank account over a period of 10 years. The account has interest rate 4%, and the deposits add up to \$100 per month. Use the future value of a continuous income stream formula (below) to estimate how much money will be in the bank account at the end of the 10 years.*

$$A=\int_{0}^{N}Ke^{r(N-t)}\,dt$$

The challenge here is to pick out each of the values to go into the equation. It is clear that $r=.04$ and $N=10$, but notice that since the deposits are about \$100 per month, the *annual* deposits (which is what K is) are \$1200, so $K=1200$. Then you set up and solve the integral:

$$A=\int_{0}^{10}1200e^{.4-.04t}$$

$$=\frac{1200}{-.04}e^{.4-.04t}\Big|_{0}^{10}=-30000(e^0-e^{.4})=30000(e^{.4}-1)$$

(b) *On a small racetrack, a car takes a rolling start and drives from point A to point B in 8 seconds. The velocity at time t seconds is given by the equation $v(t)=12t+20$ (in m/sec).*

i. *What is the average velocity of the car over its 8 seconds of travel?*

$$\frac{1}{8}\int_{0}^{8}(12t+20)\,dt,\ \text{which I trust you to integrate.}$$

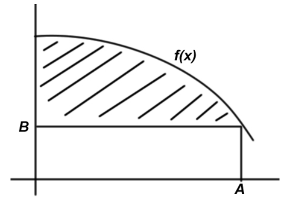
ii. *What is the total distance covered from point A to point B?*

$$\int_{0}^{8}(12t+20)\,dt,\ \text{which I trust you to integrate.}$$

(c) *Set up but do not solve an integral to determine the volume of the solid of revolution obtained by rotating the function $f(x)=(\ln(x))^5$ around the x-axis from $x=1$ to $x=5$.*

$$\int_{1}^{5}\pi(\ln(x))^{10}\,dx$$

(d) *Determine the consumer surplus for the demand function* $f(x) = \sqrt{50-x} + 145$ *at sales level* $A = 25$. *Use the graph pictured below to help you.*



$A = 25,\ B = f(25) = 150,$ so the integral to get the surplus is:

$$\int_0^{25} (\sqrt{50-x} + 145 - 150)\, dx,$$ which I trust you to simplify and then integrate.

5. (a) *Let* $f(x, y, z) = xy + xyz + xe^z$.

i. *What is* $f(2, 3, 0)$*?*

$$f(2,3,0) = 2 \cdot 3 + 0 + 2e^0 = 6 + 2 = 8$$

ii. *Find* $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, *and* $\frac{\partial f}{\partial z}$.

$$\frac{\partial f}{\partial x} = y + yz + e^z; \quad \frac{\partial f}{\partial y} = x + xz; \quad \frac{\partial f}{\partial z} = xy + xe^z$$

(b) *Draw the level curves at heights* $z = 0,\ z = 1,\ z = 2$, *for the function* $g(x, y) = 2x - y$.
The level curves are $y = 2x$, $y = 2x - 1$, and $y = 2x - 2$, which are easy to draw (all straight lines with slope 2 translated down by 1 each time). Drawing them together on the same axes is fine, as is drawing three separate graphs.