

Practice for Exam 3

Math 221 03xx and 05xx [Shaw]

No calculators allowed. You must show work at all times in order to receive full credit.

1. Find the integrals indicated below:

(a) $\int 3e^{-4x+13} dx$

(b) $\int z^3 + \frac{1}{2z^3} dz$

(c) $\int_{-4}^{-3} -\frac{3}{t} dt$

2. (a) Using left endpoints with $n = 4$, estimate the area between the curve $f(x) = 4 - x^2$ and the x -axis.

(b) Find the function $f(x)$ which has the following properties:

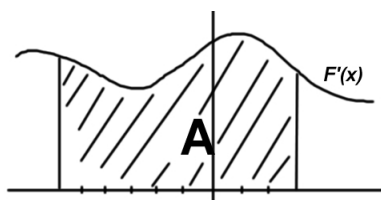
$$\begin{aligned} f''(x) &= 4e^{2x} \\ f'(1) &= 2e^2 \\ f(1) &= e^2 + 3 \end{aligned}$$

(c) Determine whether or not it is true that:

$$\int \ln(x) dx = x \ln(x) - x + C$$

3. (a) Find the area of the region enclosed by the functions $y = \sqrt{-x}$ and $y = -\frac{1}{3}x$ (your answer should be simplified).

(b) Suppose that the function $F(x)$ has the property that $F(-6) = 3$ and $F(3) = 10$. The graph of its derivative $F'(x)$ is pictured below. What is the area of the region A ?



4. (a) Suppose money is deposited daily into a bank account over a period of 10 years. The account has interest rate 4%, and the deposits add up to \$100 per month. Use the future value of a continuous income stream formula (below) to estimate how much money will be in the bank account at the end of the 10 years.

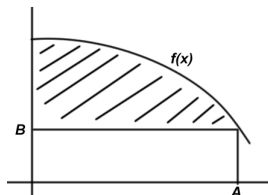
$$A = \int_0^N K e^{r(N-t)} dt$$

(b) On a small racetrack, a car takes a rolling start and drives from point A to point B in 8 seconds. The velocity at time t seconds is given by the equation $v(t) = 12t + 20$ (in m/sec).

- i. What is the average velocity of the car over its 8 seconds of travel?
- ii. What is the total distance covered from point A to point B?

(c) *Set up but do not solve* an integral to determine the volume of the solid of revolution obtained by rotating the function $f(x) = (\ln(x))^5$ around the x -axis from $x = 1$ to $x = 5$.

(d) Determine the consumer surplus for the demand function $f(x) = \sqrt{50 - x} + 145$ at sales level $A = 25$. Use the graph pictured below to help you.



5. (a) Let $f(x, y, z) = xy + xyz + xe^z$.

- i. What is $f(2, 3, 0)$?
- ii. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.

(b) Draw the level curves at heights $z = 0$, $z = 1$, $z = 2$, for the function $g(x, y) = 2x - y$.