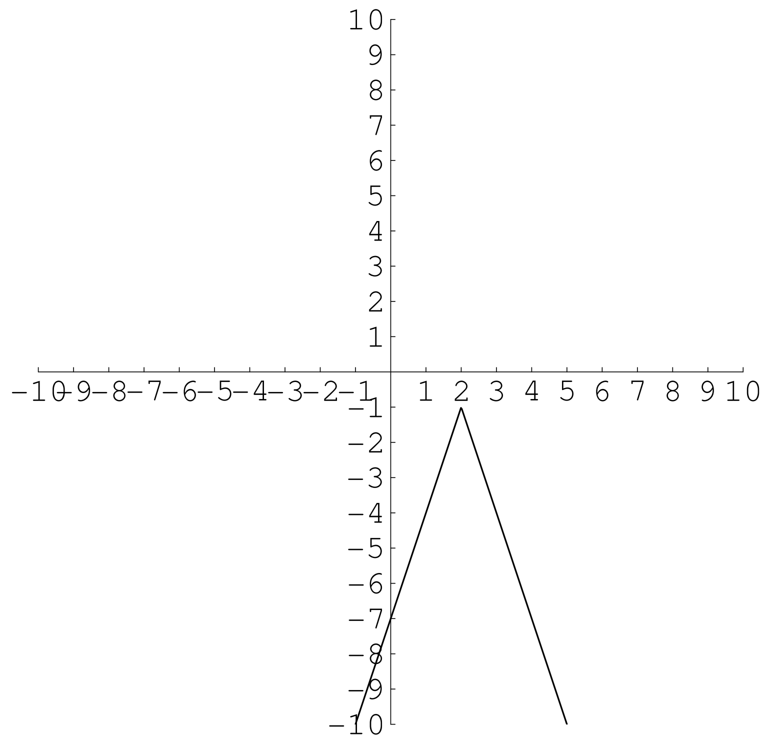
Solutions. **Exam 1. June 15, 2007.**
Math 0115 Sec 0101. Summer 2007.

[15] 1. (A) Sketch the graph of $y = -3|-x+2| - 1$. *Be sure to label your graph appropriately. (B) Using your graph, state on which intervals the function is increasing and decreasing.*

As I stated in class, I had not gone over with you the proper technique for deciphering a graph of this level of complication, so this question has been thrown out of the test. However, here is how you do it:

You must re-write the function so that the negative sign accounts for all of the inside of the absolute value: $y = -3|-(x-2)|-1$. Now, since $|-x| = |x|$, we know that $|-(x-2)| = |x-2|$, so the graph is the same as the graph of $y = -3|x-2| - 1$. In this case, we know it is a shift to the right by two, down by one, flipped upside down (by the -3) and stretched downward/shrunken horizontally. Again, this kind of question doesn't appear in your book anywhere and I didn't spend time on it in class, so yo are excused from knowing how to do it on the test.



(A) Graph of $y = -3|-x+2| - 1$
(B) Increasing on: $(-\infty, 2)$
Decreasing on: $(2, +\infty)$

[15] 2. The revenue generated from selling x units of a certain commodity is found to be $R(x) = 75x - x^2$, *where* $R(x)$ *is measured in dollars. (A) What is the maximum revenue?*

(B) How many units must be manufactured to meet this maximum? (C) Write the quadratic equation $R(x)$ in standard form.

(A) The maximum revenue is the value of R obtained when $x = \frac{-b}{2a}$, or when $x = \frac{-75}{2(-1)} = \frac{75}{2} = 37.5$, which is thus the answer for part (B). The corresponding R-value is $R(37.5) = (75)(37.5) - (37.5)^2$ dollars.

(B) 37.5 (*see above*).

(C) $-x^2+75x = (-1)(x^2-75x) = (-1)(x^2-75x+37.5^2-37.5^2) = (-1)(x^2-75x+37.5^2)+(-1)(-37.5^2) = -(x-37.5)^2+37.5^2$.

[10] 3. Find the solution set to the following inequality and express it in interval or set-builder notation: $|3x+4| > 4$.

Remember that in the case of the absolute value, you break it up into cases depending on whether you have $|a| < b$ or $|a| > b$. In this case it is the latter, so the cases are:

$$\begin{array}{cc} 3x+4>4 & 3x+4<-4 \\ 3x>4-4=0 & 3x<-4-4=-8 \\ 3x>0 & 3x<-8 \\ x>0 & x<\frac{-8}{3} \end{array}$$

So, the solution is $\{x : x < -\frac{8}{3} \text{ or } x > 0\}$, or $(-\infty, -\frac{8}{3}) \cup (0, +\infty)$.

[10] 4. Find all x which make the following true: $x = \sqrt{-2x-1}$.

To get rid of the square root, we square both sides, and then solve:

$$\begin{aligned} x^2 &= -2x-2 \\ x^2+2x+1 &= 0 \\ (x+1)^2 &= 0 \\ x &= -1 \end{aligned}$$

So the only possible answer is $x = -1$. But remember, we must always check our answers when they involve radical equations... doing so tells us that $-1 = \sqrt{-2x-1}$, which is impossible since $\sqrt{-2x-1}$ is the *positive* root. Hence there are *no* values of x which make the equation true.

[10] 5. Let $f(x) = \dfrac{1}{x^3}+1$. (A) Is f odd, even, or neither? Explain why, using the definitions of odd and even functions. (B) Is f a one-one function? If not, explain why. If so, find the inverse $f^{-1}(x)$.

(A) Neither. Plug in $x = -1$ and you get $-1+1 = 0$. Plug in $x = 1$ and you get $1+1 = 2$. Even means $f(-x) = f(x)$ and $f(-x) = -f(x)$, neither of which are the case here.

(B) f is a one-one function (draw the graph and the horizontal line test tells you this). The inverse is found by switching the variables and solving for y (all the way):

$$x = \frac{1}{y^3} + 1$$
$$x - 1 = \frac{1}{y^3}$$
$$y^3 = \frac{1}{x-1}$$
$$y = \sqrt[3]{\frac{1}{x-1}}$$

[10] 6. For each of the following, determine whether the equation describes y as a function of x. If not, say why. If so, what is the domain of the function?

$$(A) \quad x^2 = \frac{1}{3y^3 - 1} \qquad\qquad (B) \quad x^3 = \frac{1}{3y^2 - 1}$$

(A) Yes (y is to the third power). $\frac{1}{3y^3-1}$ can never be zero, so the domain is $\{x : x \neq 0\}$.

(B) No. This is because if $x = \frac{1}{2}$, y could be ± 1.

[5] 7. Find the average rate of change of the function $y = 4x^{-\frac{1}{2}} + 1$ from $x = 4$ to $x = 8$.

The average rate of change formula for $f(x)$ from $x = a$ to $x = b$ is $\frac{f(b)-f(a)}{b-a}$. In this case, that becomes:

$$\frac{(4(8^{-\frac{1}{2}}) + 1) - (4(4^{-\frac{1}{2}}) + 1)}{8 - 4} = \frac{(\frac{4}{\sqrt{8}} + 1) - (\frac{4}{\sqrt{4}} + 1)}{4} = \frac{\frac{4\sqrt{8}}{8} + 1 - 2 - 1}{4} = \frac{\sqrt{2} - 2}{4}$$

[10] 8. On a 440-mile drive from Washington, DC to Boston, MA, I stopped at a rest stop, then drove the rest of the trip at 60mph, which was 10mph faster than I had been driving before. If the entire trip took me 8 hours of driving, how much time did I spend in the car before the rest stop?

The formula to use is $D = RT$ (distance/rate/time). Let $t =$ the amount of time driving at the lower speed (50mph). Then the rest of the time would be $8 - t$, since the total trip time was 8 hours. Thus the first distance covered was $50t$ and the second was $60(8-t)$. Since the total distance was 440 miles, the formula is $440 = 50t + 60(8 - t)$. Solve that to get $t = 4$ hours.

[9] 9. Let $f(x) = \sqrt{2x + 4}$ and $g(x) = \frac{1}{x}$. Find and state the domain of:

$$(A) \quad (g - f)(x) \qquad\qquad (B) \quad (g \circ f)(x) \qquad\qquad (C) \quad \frac{f}{g}(x)$$

(A) $(g - f)(x) = \frac{1}{x} - \sqrt{2x + 4}$. Domain is the intersection of the domains, so $x \geq -2$ and $x \neq 0$, or $[-2, 0) \cup (0, +\infty)$.

(B) $(g \circ f)(x) = \frac{1}{\sqrt{2x+4}}$. Domain is the domain of f intersected with the domain of the resulting function, so $x > -2$, or $(-2, +\infty)$.

(C) $\frac{f}{g}(x) = \frac{\sqrt{2x+4}}{\frac{1}{x}} = x\sqrt{2x+4}$. Domain is again the intersection of the domains, making sure $g(x)$ nonzero (but $\frac{1}{x}$ is never zero, so we don't have to worry about it), so $x \geq -2$ and $x \neq 0$, or $[-2, 0) \cup (0, +\infty)$.

[6] 10. Give an example of each: (A) A real number that is not a rational number; (B) A rational number that is not an integer; (C) An integer that is not a natural number.

(A) $\sqrt{2}$ or π work.

(B) Any fraction that doesn't divide evenly, like $\frac{3}{4}$. Or any number with a nonzero terminating or repeating decimal that has something other than 9's in it, such as 4.6 or $0.00\overline{3}$.

(C) Any negative whole number, like -3.