The war on apathy in a terminal statistics course: Motivating definitions from day one

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- Forget and move on

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- In an ideal world, could ask questions which measure understanding, not memorization.
- Necessary first step: proper motivation of concepts.
  - Lead students to come up with definitions on their own.

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- Average: motivate the different occasions for using mean, median, and mode.
- Continuous vs. discrete random variables.

Survey Questions

1. Guess your professor's age, in years.
2. What is your height in feet, to the nearest thousandth?
3. How many tattoos do you have?
4. What is your class in school?

Survey Results

1. My age (years)	2. Height (feet)	3. Tattoos	4. Class
26	5.417	0	Freshman
28	5.083	1	Freshman
29	5.083	0	Sophomore
30	6.041	0	Freshman
31	5.5	0	Freshman
37	5.083	0	Junior
36	5.833	2	Senior
28	5.75	1	Freshman
27	5.5	0	Sophomore
36	5.917	1	Sophomore
35	5.958	1	Freshman
30	5.875	0	Freshman
31	5.25	1	Sophomore
32	5.833	4	Senior
30	5.167	0	Freshman
31	6	0	Freshman
36	5.917	0	Freshman
25	5.583	0	Sophomore
24	5.333	0	Freshman
29	5.333	1	Freshman
30	5.75	2	Sophomore
31	5.167	1	Junior
32	5.875	0	Freshman

Fill in the Blanks

- The average person in this class _____
- On average, students in this class _____
- The average result for the survey in this class is _____

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- Unanimity makes a good argument that any of these could work as the *average*.

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- Also happens to be the minimum value; as such, should not be in contention for the average.

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- Not meant to fit into the 'average' sentences given above; better to choose median or mode.

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- *Freshman* $= 0$, *etc.*

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- X is more than just a single number; X carries the probabilistic properties of its possible values.

Random Variables and Sample Spaces

Number of Tattoos	Count	Probability
0	13	13/23
1	7	7/23
2	2	2/23
4	1	1/23

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  - $P(Y = 6) \approx 0$
  - Better: $P(5.95 \leq Y \leq 6) = ?$

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- These two examples are fundamentally different.
- Have students formulate definitions for *discrete* and *continuous* random variables.

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- Levity and humor