

Unifying sets and logic with the real world in a Liberal Arts Mathematics course

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Class environment

- 1–2 individual sections of a Liberal Arts Mathematics course within a context of 6–15 total sections each semester
- Uniform syllabus/midterm/final, HW/quizzes/exams
- Material includes general problem-solving, set algebra, symbolic deductive reasoning (quantifier-free propositional logic), pre-algebra, counting, probability
- Textbook curriculum is focused essentially on evaluation of syllogisms through Venn/Euler diagrams.

Learning outcomes evinced by a typical text

- Evaluate the logical cause/effect structure of complex language. (Translate & use a truth table)
- Determine appropriate logical conclusions that can be drawn from given information. (Complete a syllogism)
- Evaluate the logical validity of conclusions drawn from given information. (Evaluate a syllogism)

Framing questions

- What will a student use deductive reasoning for outside of mathematics?
- Do students recognize the usefulness of this work in their lives?
- Are there contexts in which logical validity exists at odds with conventional intuition?
- Are syllogisms and truth tables effective in seeding those skills?

Basic (type A) translating/parsing exercises

Write the statement in symbolic form.

Every house on this block has a mailbox.

Let a = "The animal is a lizard," and

b = "The animal has a tail."

Translate the sentence $\sim b \rightarrow \sim a$ to plain English.

What is the truth value of the statement, $p \rightarrow (q \vee \sim r)$, given that p and r are false and q is true?

Context matters

- Hypothetical / abstract / Lewis Carroll-style exercises are easy to come up with.
- The language of laws, taxes, and contracts are often heavy with adjectives, conditions, and clauses, but with their Boolean structures are rare examples of a naturally-occurring direct application.

Motivated (type B) translating exercises

Write the rule that is being described by each sign in the form of a logical statement, defining simple sentences with letters and using the appropriate symbols.



Type B parsing exercises

For each sign below, describe a circumstance that would lead to a driver acting in violation of the rule being described.



Type A evaluation of statements

Consider the given statements:

If a person is shivering, then they are cold.

If a person is cold, then they are not wearing a coat.

What is a logical conclusion that can be drawn from these two statements?

Consider the given axiomatic statements:

All ants are insects.

No insects have more than six legs.

Is it valid to conclude that, "No ants have more than six legs."?

Type B evaluation of statements

You have been empaneled for a jury that is considering evidence in a trial for Mark, who is accused of burglarizing a restaurant. The court allows testimony that supports an accusation that the defendant had previously burglarized a home, for which the defendant was never prosecuted. A judge presents you with the following instructions for you to use when evaluating the evidence.

I instruct you that before you may consider any other transaction for the limited purpose I have referred to, you must first determine whether this accused committed the other transaction, and if so, whether any such act was similar enough to the crime charged in this indictment, so that proof of another transaction in light of the limited purpose for which it has been presented to you would tend to prove an element of a crime charged in this indictment.

In your own words, what must you as a juror take into consideration when evaluating whether or how to consider the testimony about the home burglary?

Can classwork focused on Type A questions impact understanding for Type B questions?

- Measured student understanding using pre- and post-tests in several sections of LAM course ($n = 34$).
- Pre-test prior to unit in class, post- test one week after unit completed. Each test contained several Type A and Type B questions. Open-ended questions evaluated qualitatively as some answers could be incomplete.

Results

| | | |
|-------|--------|--------------|
| A Pre | A Post | A Difference |
| 39.7% | 62.1% | 22.4% |
| B Pre | B Post | B Difference |
| 35.3% | 34.5% | -0.8% |

Informal results, but they point toward the need for earlier/deeper/better implementation of Type B work in classroom exercises.

Thanks, and questions?